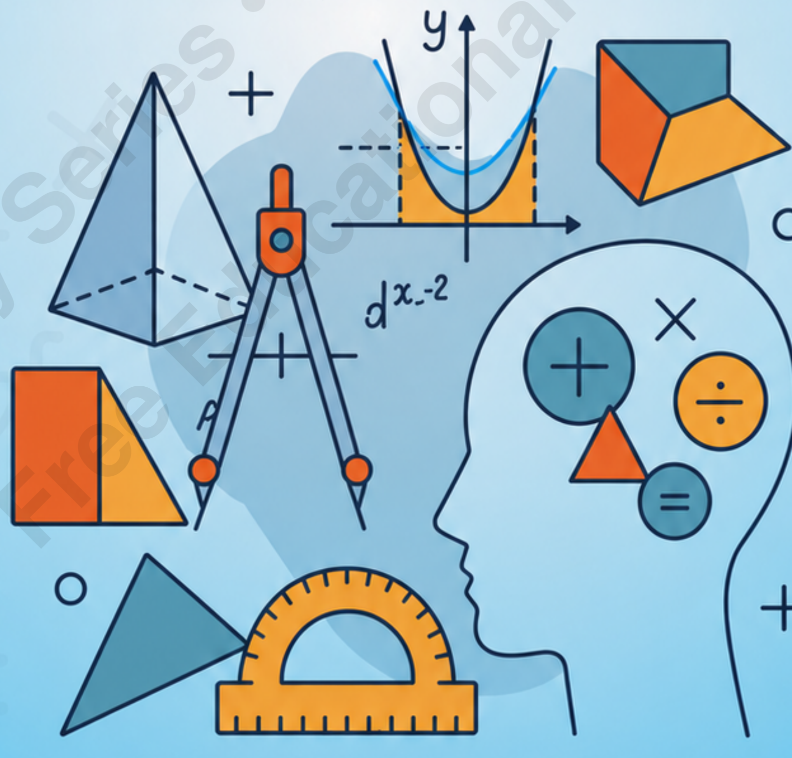
Preppy Series

MATHEMATICS

for Junior High Schools

BECE Past Questions & Solutions

Arranged by Year



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Mathematics

for Junior High Schools

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Compiled by

preppyonline.org

2024 Edition

Preface

This book is designed to be a helpful resource for junior high school students, especially those preparing for the Basic Education Certificate Examination (BECE) of the West African Examinations Council (WAEC).

It features

- a collection of complete BECE Mathematics past papers—both objective and essay-type questions—arranged by year
- detailed and thoughtfully explained solutions
- short notes on important facts and methods frequently used on the exams

Our philosophy is simple: learning Mathematics should go beyond memorizing procedures or blindly applying algorithms and tricks. True understanding comes from engaging with problems, exploring different approaches, and appreciating how and why solutions work. For this reason, each solution is presented

- in a step by step fashion
- with clear reasoning
- with multiple perspectives where appropriate
- and with an emphasis on building strong conceptual foundations

Special attention has been given to clarity, with well-drawn diagrams and structured workings that carry the student along through the solution process.

In line with the goal of widening access to quality educational materials, this resource is available at preppyonline.org as a free PDF download. It is our hope that students, teachers, and independent learners alike will find it useful not just for exam preparation, but for developing confidence and deeper insight into Mathematics.

Comments, suggestions, and other enquiries may be sent to preppyonline@gmail.com

Contents

Chapter 1

2024 Paper 1

1. Two brands of air conditioners S and T cost GH¢ 3,000.00 and GH¢ 4,000.00 respectively. A company budgeted GH¢ 20,000.00 to buy air conditioners. If the company bought 5 units of brand S instead of brand T, how much did it save?

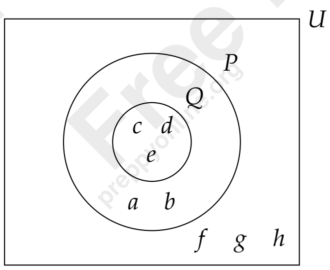
A. GH¢ 1,000.00
B. GH¢ 5,000.00
C. GH¢ 20,000.00
D. GH¢ 15,000.00

2. Zalia and Amina shared an amount of money in the ratio 2 : 5. If Amina had GH¢ 150.00 **more** than Zalia, how much did they share?

A. GH¢ 100.00
B. GH¢ 250.00
C. GH¢ 450.00
D. GH¢ 350.00

3. Which of the following is an example of quantitative data?

A. Colour
B. Gender
C. Marital status
D. Length



In the diagram, P and Q are two sets and U is the universal set.

Use the information to answer questions 4 and 5.

4. Find $P \cap Q$.

A. $\{c, d, e\}$
B. $\{a, b\}$
C. $\{a, b, c, d, e\}$
D. $\{f, g, h\}$

5. How many members are in set Q?

A. 2
B. 3
C. 8
D. 5

6. Amadu walked to a point such that he is always the same distance from two villages P and Q. Which of the following best describes the locus of Amadu?

A. An arc passing through line PQ
B. A circle passing through line PQ
C. Straight line PQ
D. Perpendicular bisector of line PQ

7. A bag of rice weighs 2 kg. If the empty bag weighs 150 g, find the weight of the rice. [1 kg = 1,000 g]

A. 0.175 kg
B. 0.185 kg
C. 1.850 kg
D. 1.750 kg

8. Andrews drew three lines such that the length of the first one is 10 cm, the second is 15 cm longer than the first one and the third is 9 cm less than the second. Find the length of the third line.

A. 4 cm
B. 14 cm
C. 34 cm
D. 16 cm

9. Find the truth set of $2x - 4 < 6 + 3x$.

A. $\{x : x < 2\}$
B. $\{x : x > -2\}$
C. $\{x : x > -10\}$

D. $\{x : x < 10\}$

10. The locus of points equidistant from a fixed point is called a

A. chord.

B. circle.

C. mediator.

D. diameter.

11. Evaluate: $\sqrt{75} + \sqrt{18} - \sqrt{27}$.

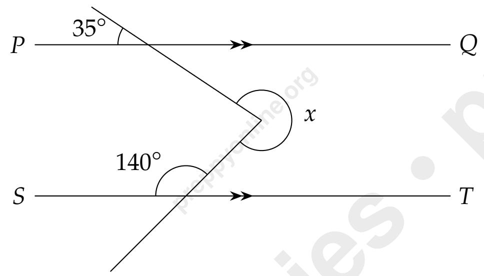
A. $2\sqrt{3} - 3\sqrt{2}$

B. $2\sqrt{3} + 3\sqrt{2}$

C. $\sqrt{6}$

D. $5\sqrt{6}$

12.



NOT DRAWN TO SCALE

In the diagram, line PQ is parallel to line ST. Find the value of the angle marked x.

A. 140°

B. 220°

C. 290°

D. 285°

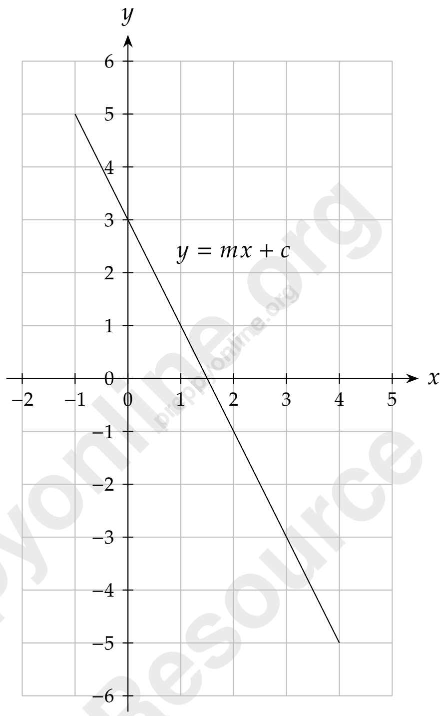
13. A hawker is carrying a basket load of three types of fruits: oranges, mangoes and pears. If $\frac{2}{5}$ of the fruits are oranges and $\frac{6}{25}$ mangoes, what percentage of the fruits are pears?

A. 9%

B. 18%

C. 64%

D. 36%



The diagram shows the graph of a linear relation of the form $y = mx + c$.

Use the graph to answer questions 14 and 15.

14. Find the slope of the relation.

A. −2

B. −1

C. 2

D. 1

15. Find the equation of the relation.

A. $y = -2x + 3$

B. $y = -2x + 2$

C. $y = 2x + 3$

D. $y = 2x - 3$

16. Araba is 3 years younger than her sister. If the sum of their ages is 17 years, find Araba's age.

A. 7 years

B. 8 years

C. 10 years

D. 9 years

17. A number of oranges are shared among 50 students and **each** got 15 oranges. If the same number of oranges are shared equally among 30 students, how many will **each** get?

A. 13

B. 15

C. 25
D. 20

18. If the bearing of Q from P is 120°, find the bearing of P from Q.
A. 060°
B. 210°
C. 300°
D. 240°

19. Given the vectors $\mathbf{m} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ and $\mathbf{n} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$, find $2\mathbf{m} + \mathbf{n}$.
A. $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$
B. $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$
C. $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$
D. $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$

20. Simplify: $\frac{27^{m+1}}{3^{m+2}}$.
A. 3^{2m}
B. 3^{2m-1}
C. 3^{2m+2}
D. 3^{2m+1}

21. There are 15 white and 25 black identical balls in a box. If a ball is selected at random from the box, find the probability that it is white.
A. $\frac{1}{25}$
B. $\frac{1}{15}$
C. $\frac{3}{8}$
D. $\frac{5}{8}$

22. The area of a rectangle is 18 cm^2. If the width is 2 cm, find its perimeter.
A. 18 cm
B. 20 cm
C. 36 cm
D. 22 cm

23. Find the interest on GH¢ 400.00 for 2 years at 10% simple interest per annum.
A. GH¢ 8.00
B. GH¢ 40.00
C. GH¢ 80.00
D. GH¢ 60.00

24. One of the factors of the expression $4m^2 + 12m - 8m - 24$ is $(4m - 8)$. Find the other factor.
A. $m - 3$
B. $m + 3$
C. $2m + 3$
D. $2m - 3$

25. An article which cost GH¢ 600.00 was sold at a discount of 10%. Find the selling price.
A. GH¢ 60.00
B. GH¢ 504.00
C. GH¢ 560.00
D. GH¢ 540.00

26. Make m the subject of the relation $\frac{1}{m} = \frac{1}{p} + \frac{1}{r}$.
A. $m = \frac{pr}{r+p}$
B. $m = \frac{pr}{r-p}$
C. $m = \frac{r-p}{pr}$
D. $m = \frac{r+p}{pr}$

27. The point $P(-2, 3)$ is translated by a vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ to a point R. Find the coordinates of R.
A. $(6, -2)$
B. $(-3, 6)$
C. $(-3, -6)$
D. $(-1, 0)$

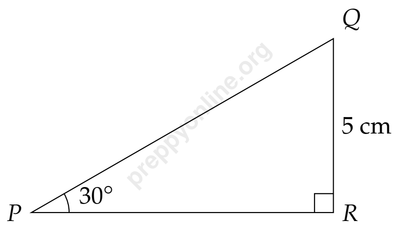
28. A frog leaps in such a way that its distance, in metres, from its starting position after **each** leap is given by 4, 7, 10, ... Find its distance from the starting position after the 10th leap.
A. 28
B. 31
C. 37
D. 34

29. A bus departed from Elmina at 9:15 pm and arrived in Accra at 2:45 am the next day. How long did the journey take?
A. 4 hours 20 minutes
B. 4 hours 30 minutes
C. 5 hours 30 minutes
D. 5 hours 20 minutes

30. Simplify: $3x - 2(3 + 2x) + x(2x + 4)$.
A. $2x^2 + 11x - 6$

B. $2x^2 + 3x - 6$
C. $2x^2 - 4x - 6$
D. $2x^2 + 4x - 6$

31.



NOT DRAWN TO SCALE

In the diagram, $\angle QPR = 30°$ and $|QR| = 5$ cm. [Take $\sin 30° = \frac{1}{2}$].

Find the length of $\overline{PQ}$.

A. 2.5 cm
B. 5.0 cm
C. 12.0 cm
D. 10.0 cm

32. A fair coin and a fair die are rolled together **once**. Find the probability of obtaining a tail and an odd number.

A. $\frac{1}{4}$
B. $\frac{1}{3}$
C. $\frac{2}{3}$
D. $\frac{1}{2}$

33. If the gradient of a straight line is zero, then the line

A. is vertical.
B. is horizontal.
C. falls to the right.
D. rises to the right.

34. Madam Nancy wants to know which of the teachers in her school is liked best by most of the students. Which of the following methods is **most suitable** for collecting the data?

A. Experiment
B. Database
C. Questionnaire
D. Observation

35. Mansah packed 1,800 apples into a number of boxes. If **each** box contained 120 apples, how many boxes were fully packed?

A. 15
B. 16
C. 18
D. 17

36. The cost of three items at a shop are GH¢ 72.00, GH¢ 1,105.00 and GH¢ 216.00. If a customer bought all the three items and received a change of GH¢ 107.00, how much did he initially give the shopkeeper?

A. GH¢ 1,300.00
B. GH¢ 1,400.00
C. GH¢ 2,000.00
D. GH¢ 1,500.00

Number on die	1	2	3	4	5	6
Frequency	4	3	3	2	3	5

The table shows the results when a student tossed a die many times.

Use the information to answer questions 37 and 38.

37. Find the mode.

A. 6
B. 5
C. 3
D. 4

38. How many times did the student throw the die?

A. 6
B. 18
C. 21
D. 20

39. Find the image of the point $(-3, 5)$ when it is rotated through 360° about the origin.

A. $(5, -3)$
B. $(-3, 5)$
C. $(-5, 3)$
D. $(-3, -5)$

40. A story book contains 50 pages. If a student reads 10 pages per hour, find the relationship between the number of **unread** pages (N) and time (t).

A. $N = 10t + 50$
B. $N = -10t + 50$
C. $N = -\frac{1}{10}t + 50$
D. $N = 10t - 50$

Chapter 2

Solutions to 2024 Paper 1

Answer key

1. B	9. C	17. C	25. D	33. B
2. D	10. B	18. C	26. A	34. C
3. D	11. B	19. A	27. B	35. A
4. A	12. D	20. D	28. B	36. D
5. B	13. D	21. C	29. C	37. A
6. D	14. A	22. D	30. B	38. D
7. C	15. A	23. C	31. D	39. B
8. D	16. A	24. B	32. A	40. B

Solutions

1. Answer: B

The company wanted to buy some air conditioners and had the option of buying brand S (price: GH¢ 3,000.00) or brand T (price: GH¢ 4,000.00).

Because brand S was cheaper than brand T, the company saved some money whenever it bought brand S instead of brand T.

For each brand S air conditioner the company bought instead of brand T, it saved

$$\text{GH¢}\ 4,000.00 - \text{GH¢}\ 3,000.00 = \text{GH¢}\ 1,000.00$$

Since the company bought 5 units of brand S instead of brand T, the total amount the company saved was

$$5 \times \text{GH¢}\ 1,000.00 = \text{GH¢}\ 5,000.00$$

It is interesting that the examiners mentioned a budget of GH¢ 20,000.00 in the question even though we did not need that information to answer it. While it is true that a budget of GH¢ 20,000.00 could only buy 5 units of an air conditioner that cost GH¢ 4,000.00 each, we didn't need that information to solve the problem. Sometimes questions include information that is not necessary to solve them. However, in such situations, we should be careful not to get distracted by the irrelevant information.

2. Answer: D

Even though we are not told the amount of money Zalia and Amina shared, we are told the ratio in which they shared it: 2 : 5.

From this ratio, we know the fraction of the total amount shared that each of them received.

The fraction of the total amount that Zalia received was

$$\frac{2}{2+5} = \frac{2}{7}$$

And the fraction of the total amount that Amina received was

$$\frac{5}{2+5} = \frac{5}{7}$$

Thus, Amina got more money that Zalia. The difference between the amount Amina got and what Zalia got was

$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7} \text{ of the total amount shared}$$

From the question, this amount was GH¢ 150.00.

Letting A be the total amount of money they shared, we have

$$\frac{3}{7}A = \text{GH¢}\ 150$$

We can then solve this equation to get the amount of money they shared, A.

$$\begin{aligned}\frac{3}{7}A &= \text{GH¢}\ 150\\ 3A &= \text{GH¢}\ 150 \times 7\\ A &= \text{GH¢}\ \frac{150 \times 7}{3}\\ A &= \text{GH¢}\ 350.00\end{aligned}$$

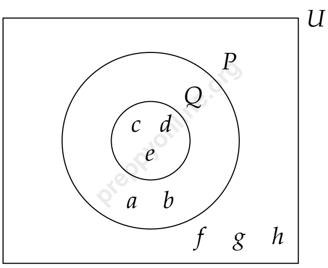
3. Answer: D

Quantitative data are values that can be measured numerically (Section 5.17.1).

Length is quantitative as it can be measured in numbers like metres, centimetres, etc.

All the other options are qualitative as they describe qualities or attributes rather than numerical measurements.

4. Answer: A



$P \cap Q$ is the intersection of the sets P and Q. The elements of this set are those that are in both P and Q. From the Venn diagram, these elements are c, d, e.

Hence, $P \cap Q = \{c, d, e\}$.

Note that, though a and b are in P, they are not in Q. Hence, they cannot be in $P \cap Q$.

f, g, and h also cannot be in $P \cap Q$ as they are neither in P nor in Q.

5. Answer: B

The members of set Q are those that are in the circle labeled Q in the given Venn diagram. These are c, d, and e.

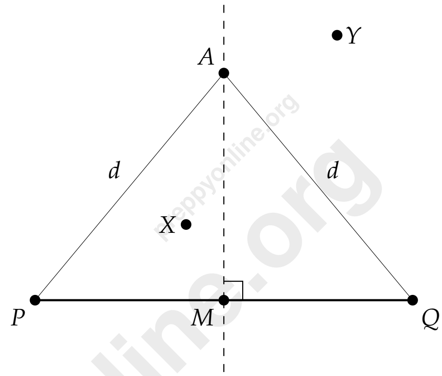
Hence, set Q has three members.

6. Answer: D

If Amadu is always the same distance from village P as he is from village Q, his path traces the perpendicular bisector of PQ.

This perpendicular bisector is drawn with a broken vertical line in the sketch below.

Perpendicular bisector



To convince ourselves of the fact that all points that are the same distance from P and Q must be on the perpendicular bisector of PQ, we may pick some arbitrary points in the sketch and verify that they are closer to P than Q or vice versa.

Consider point X.

Notice that it is not on the perpendicular bisector. Which of the points P or Q is it closer to? You may use a rule, a pair of compasses, or a pair of dividers to check this.

What about the point Y? Is it closer to P or Q?

On the contrary, consider point A. Notice that it lies on the perpendicular bisector. What is its distance from P? And what is its distance from Q?

What about point M?

Challenge: Every point that is not on the perpendicular bisector is not the same distance from P as it is from Q. Try to check this fact with a rule, a pair of compasses, or a pair of dividers.

Another challenge: Every point on the perpendicular bisector is the same distance from both P and Q.

7. Answer: C

We are given the weight of a bag of rice as 2 kg. But this weight includes both the weight of the bag that holds the rice and rice itself, which is the contents of the bag.

Since the empty bag weighs 150 g, the weight of the rice in the bag is

$$2 \text{ kg} - 150 \text{ g}$$

Since 1 kg = 1,000 g, 2 kg = 2,000 g and we may write the weight of the rice as

$$\begin{aligned} 2 \text{ kg} - 150 \text{ g} &= 2000 \text{ g} - 150 \text{ g} \\ &= 1850 \text{ g} \end{aligned}$$

Since all the answer options given are in kilograms, we would like to express the answer above in kilograms as follows:

$$\begin{aligned}1850 \text{ g} &= \frac{1850}{1000} \text{ kg}\\ &= 1.850 \text{ kg}\end{aligned}$$

8. Answer: D

We are given the lengths of three lines Andrews drew and given the relationships between these lengths. We are to use these relationships to deduce the length of the third line.

The first line was 10 cm long.

The second line was 15 cm longer than the first line. That means that the length of the second line was

$$10 \text{ cm} + 15 \text{ cm} = 25 \text{ cm}$$

The third line was 9 cm shorter than the second line. That means that the length of the third line was

$$25 \text{ cm} - 9 \text{ cm} = 16 \text{ cm}$$

9. Answer: C

To find the truth set of the inequality, we solve it to get all the values of x for which the inequality is satisfied.

$$\begin{aligned}2x - 4 &< 6 + 3x\\ 2x - 3x &< 6 + 4\\ -x &< 10\end{aligned}$$

To isolate x, we may multiply or divide both sides of the inequality by -1. However, we should not forget that doing so reverses the inequality.

$$-x < 10 \quad \text{means that} \quad x > -10$$

10. Answer: B

Given a fixed point, if a path is such that its points are always the same distance from the fixed point, the path must be a circle (Section 5.18).

11. Answer: B

We are expected to use the properties of surds (Section 5.10) to evaluate the given expression.

$$\begin{aligned}\sqrt{75} + \sqrt{18} - \sqrt{27} &= \sqrt{25 \cdot 3} + \sqrt{9 \cdot 2} - \sqrt{9 \cdot 3}\\ &= \sqrt{25}\sqrt{3} + \sqrt{9}\sqrt{2} - \sqrt{9}\sqrt{3}\\ &= 5\sqrt{3} + 3\sqrt{2} - 3\sqrt{3}\\ &= 2\sqrt{3} + 3\sqrt{2}\end{aligned}$$

12. Answer: D

There are many ways to approach this problem. However, all of them boil down to understanding the relationship between angles in parallel lines.

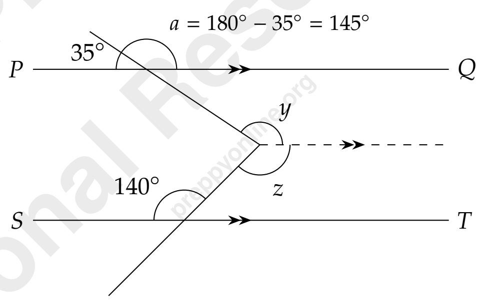
If we manage to break angle x into smaller parts and figure out the value of each of those parts, we can then combine the parts together to get the measure of the full angle.

We shall explore two ways of solving the problem. Try to come up with your own methods too.

Method 1

We add a broken line parallel to both PQ and ST to help us solve the problem. This line divides angle x into two parts, y and z, so that

$$x = y + z$$



Angle a and the angle labeled 35° are angles on a straight line. Hence, they add up to 180°, and $a = 180° - 35° = 145°$.

Since angles a and y are corresponding angles, they are equal. Hence, $y = 145°$.

Angle z and the angle labeled 140° are alternate angles. Hence, they are equal. That is, $z = 140°$.

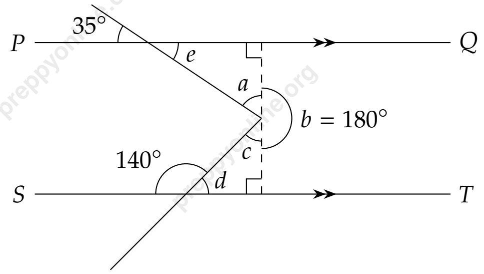
If $y = 145°$ and $z = 140°$, then we can find x as follows:

$$\begin{aligned}x &= y + z\\ &= 145° + 140°\\ &= 285°\end{aligned}$$

Method 2

We add a broken line perpendicular to both PQ and ST to help us solve the problem. This line divides angle x into three parts, a, b and c, so that

$$x = a + b + c$$



Angle b is a straight angle (Section 5.16.1) so it measures $180°$.

Angle e and the angle labeled $35°$ are vertically opposite angles so they are equal. Thus, $e = 35°$.

Angle d and the angle labeled $140°$ are angles on a straight line. Hence, they add up to $180°$, and $d = 180° - 140° = 40°$.

Angles a and e are acute angles in a right-angled triangle (Section 5.16.1) so they add up to $90°$. Since $e = 35°$,

$$a = 90° - 35° = 55°$$

Similarly, angles c and d are acute angles in a right-angled triangle so they add up to $90°$. Since $d = 40°$,

$$c = 90° - 40° = 50°$$

Since we know a, b, and c, we can find x as follows:

$$\begin{aligned} x &= a + b + c \\ &= 55° + 180° + 50° \\ &= 285° \end{aligned}$$

13. Answer: D

Since each of the fruits the hawker is carrying is either an orange, a mango, or a pear, if we add the fraction of fruits that are oranges to the fraction that are mangoes and the fraction that are pears, we should get 1.

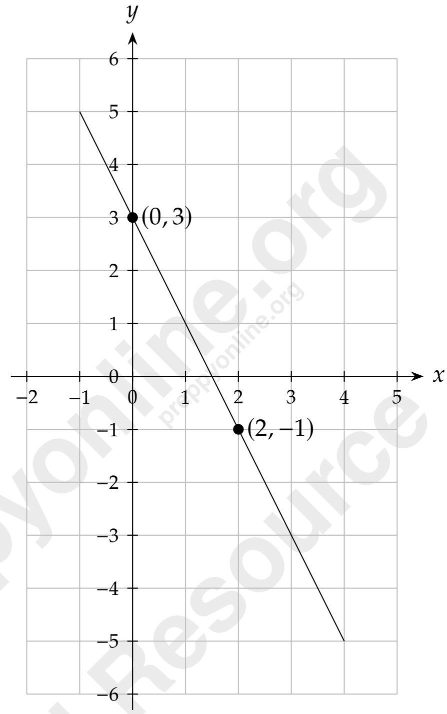
Since $\frac{2}{5}$ of the fruits are oranges and $\frac{6}{25}$ are mangoes, the fraction of the fruits that are pears is

$$\begin{aligned} 1 - \frac{2}{5} - \frac{6}{25} &= \frac{25 - 5(2) - 6}{25} \\ &= \frac{25 - 10 - 6}{25} \\ &= \frac{9}{25} \end{aligned}$$

As a percentage, this fraction is equal to

$$\frac{9}{25} \times 100\% = \frac{9}{\cancel{25}_1} \times \cancel{100}^{4}\% = 36\%$$

14. Answer: A



The formula for the slope of a relation (Section 5.12.1) is

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

To find the slope of the relation, we pick two points on the line and calculate the difference between their y coordinates and divide it by the difference between their x coordinates.

From the diagram, the points $(0, 3)$ and $(2, -1)$ are both on the line.

Computing the ratio for their slope gives

$$\begin{aligned} \text{slope} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{3 - (-1)}{0 - 2} \\ &= \frac{4}{-2} \\ &= -2 \end{aligned}$$

15. Answer: A

To find the equation of the relation, we may use the slope and a point on the line.

We have already computed the slope above as -2.

From the diagram, the y intercept (Section **??**) of the relation is 3.

If a relation has a slope of −2 and a y intercept of 3, then its equation (Section 5.12.2) is

$$y = -2x + 3$$

16. Answer: A

Let Araba's age be a.

Since Araba is three years younger than her sister, her sister is three years older than her. That means her sister's age is

$$a + 3$$

If the sum of their ages is 17, we have

$$a + a + 3 = 17$$

Solving this equation gives

$$\begin{aligned} a + a + 3 &= 17 \\ 2a + 3 &= 17 \\ 2a &= 17 - 3 = 14 \\ a &= \frac{14}{2} \\ a &= 7 \end{aligned}$$

Thus, Araba is 7 years old.

17. Answer: C

If by sharing the oranges equally among 50 students each student gets 15 oranges, then the total number of oranges shared was

$$50 \times 15 = 750$$

If we share these 750 oranges equally among 30 students, how many will each get?

$$\frac{750}{30} = 25$$

18. Answer: C

We will solve this problem in two ways.

In the first one, we have to recall an important fact about bearings and back bearings (Section 5.13). However, in the second method, we derive the back bearing from scratch, using the given information.

Method 1

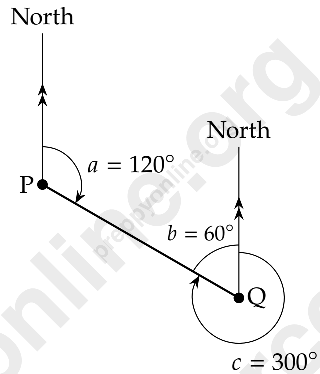
If we recall that given the bearing of Q from P, we can find the bearing of P from Q, also known as the back bearing, as follows:

If the given bearing is less than 180°, add 180° to it; if it is greater than or equal to 180°, subtract 180° from it,

then we can simply add 180° to the given bearing of 120° to get a back bearing of 300°.

Method 2

Bearings are measured clockwise from the north.



The bearing of Q from P is 120°. This is represented by angle a in the diagram.

In the diagram, the vertical lines from P and Q are parallel. This makes angles a and b co-interior angles.

Since co-interior angles add up to 180°, angle b is equal to $180° - 120° = 60°$.

The bearing of P from Q is represented by angle c.

Since angles c and b are angles at a point, they add up to 360°. Thus, since $b = 60°$, c must be equal to

$$360° - 60° = 300°$$

19. Answer: A

We are asked to evaluate an expression involving vectors. This requires us to use the rules of vector algebra (Section 5.14).

Given the vectors $\mathbf{m} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ and $\mathbf{n} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$,

$$\begin{aligned} 2\mathbf{m} + \mathbf{n} &= 2\begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 2(5) \\ 2(-1) \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 10 \\ -2 \end{pmatrix} + \begin{pmatrix} -4 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 10-4 \\ -2+2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \end{aligned}$$

20. Answer: D

We are given an expression involving indices and have to apply the rules of indices (Section 5.8) to simplify it.

Since $27 = 3^3$, the given expression may be written as

$$\begin{aligned}\frac{27^{m+1}}{3^{m+2}} &= \frac{3^{3(m+1)}}{3^{m+2}}\\ &= \frac{3^{3m+3}}{3^{m+2}}\\ &= 3^{(3m+3)-(m+2)}\\ &= 3^{3m+3-m-2}\\ &= 3^{2m+1}\end{aligned}$$

21. Answer: C

Since the balls are identical up to colour, the probability of picking a certain colour of ball is the proportion of the total number of balls that are that colour.

Thus, the probability that a ball selected at random from the box is white is

$$\begin{aligned}\frac{\text{number of white balls in the box}}{\text{total number of balls in the box}} &= \frac{15}{15+25}\\ &= \frac{15}{40}\\ &= \frac{3}{8}\end{aligned}$$

22. Answer: D

The area of a rectangle is given by

$$\text{area} = \text{length} \times \text{width}$$

Substituting the given values into this formula gives

$$18\text{ cm}^2 = \text{length} \times 2\text{ cm}$$

From this, we derive the length as

$$\text{length} = \frac{18}{2} = 9\text{ cm}$$

The perimeter of a rectangle is given by

$$\text{perimeter} = 2 \times (\text{length} + \text{width})$$

Substituting the given values into this formula gives the perimeter of the given rectangle as

$$2(9+2) = 2(11) = 22\text{ cm}$$

23. Answer: C

The formula for simple interest (Section 5.4) is

$$\text{simple interest} = \text{principal} \times \text{rate} \times \text{time}$$

We are given a principal of GH¢400.00,
a rate of 10% per annum,
and a time of 2 years.

Substituting these into the formula gives the simple interest as

$$\begin{aligned}\text{principal} \times \text{rate} \times \text{time} &= \text{GH¢ } 400 \times 10\% \times 2\\ &= \text{GH¢ } 400 \times \frac{10}{100} \times 2\\ &= \text{GH¢ } 80.00\end{aligned}$$

24. Answer: B

Method 1

The expression $4m^2 + 12m - 8m - 24$ is a quadratic in m whose constant term is -24. Thus, when it is factorized into two factors, the product of the constant terms in the two factors must be -24.

Given that one of the factors is $(4m - 8)$, the constant term in the other factor must be $+3$ as $\frac{-24}{-8} = +3$. Hence, the other factor must have the form

$$\text{something} + 3$$

Also, since the quadratic term in the expression is $4m^2$, if one of the factors is $(4m - 8)$, the other factor must be such that its term in m multiplied by $4m$ equals $4m^2$. Thus, the term in m in the other factor must be

$$\frac{4m^2}{4m} = m$$

If the constant term in the other factor is $+3$ and the term in m is m, then the other factor must be

$$m + 3$$

Method 2

Since we know how to factorize quadratic expressions, we may just go ahead and factorize the given expression from scratch, ignoring the hint given in the question.

Factorizing the expression gives

$$\begin{aligned}4m^2 + 12m - 8m - 24 &= (4m^2 + 12m) - (8m - 24)\\ &= 4m(m+3) - 8(m+3)\\ &= (m+3)(4m-8)\end{aligned}$$

Since $(4m - 8)$ is already given, the other factor is $(m + 3)$.

25. Answer: D

If the item was sold at a discount of 10%, then the discount on the item was 10% of the cost price. Thus, the discount was

$$10\% \text{ of GH¢ } 600 = \frac{10}{100} \times \text{GH¢ } 600 = \text{GH¢ } 60.00$$

And the selling price was

$$\text{GH¢ } 600 - \text{GH¢ } 60 = \text{GH¢ } 540.00$$

26. Answer: A

To make m the subject of the relation $\frac{1}{m} = \frac{1}{p} + \frac{1}{r}$, we begin by writing the right-hand side as a single fraction:

$$\frac{1}{m} = \frac{1}{p} + \frac{1}{r}$$
$$\frac{1}{m} = \frac{r+p}{pr}$$

We then cross-multiply and simplify to get

$$pr = m(r+p)$$
$$m = \frac{pr}{r+p}$$

27. Answer: B

To translate a point by a vector, we add the translation vector to the position vector of the point.

This is like adding the components of the vector to the coordinates of the given point.

Doing so, the x coordinate of the translated point is

$$-2 - 1 = -3$$

And the y coordinate of the translated point is

$$3 + 3 = 6$$

Thus, the translated point, R, is $(-3, 6)$.

28. Answer: B

We are given a linear sequence or arithmetic progression (Section 5.11.1) and asked to find its 10th term.

The frog's leaps can be represented by the figure below:

$$4 \xrightarrow{+3} 7 \xrightarrow{+3} 10 \xrightarrow{+3} 13 \xrightarrow{+3} \cdots$$

To find its distance from the starting position after 10 leaps, we may observe the pattern by which its distance from the starting position is increasing. We may also continue the pattern till we reach the 10th term.

We shall explore both of these methods in our solutions.

Method 1

Starting from a distance of 4 metres from the frog's starting position, each leap adds a distance of 3 metres to its distance from the starting position as shown below:

Leap no.	Distance from start
1	4
2	$4 + 3 = 7$
3	$4 + 3 \times 2 = 10$
4	$4 + 3 \times 3 = 13$
5	$4 + 3 \times 4 = 16$
⋮	⋮
n	$4 + 3 \times (n-1)$

Looking at the first few leaps, we observe the pattern that after leap n, the frog's distance from the starting position is given by

$$4 + 3 \times (n-1)$$

Hence, after the 10th leap, its distance from the starting position will be

$$4 + 3 \times (10-1) = 4 + 3 \times 9 = 4 + 27 = 31 \text{ metres}$$

Using this method, can you find the frog's distance from the starting position after leaping 101 times? Verify that the answer is 304 metres.

Can you find its distance from the starting position after leaping 1,001 times? Verify that the answer is 3,004 metres.

Method 2

We may continue the pattern the frog's leaps are following to see where it will land after the 10th leap:

$$4 \to 7 \to 10 \to 13 \to 16 \to 19 \to 22 \to 25 \to 28 \to 31$$

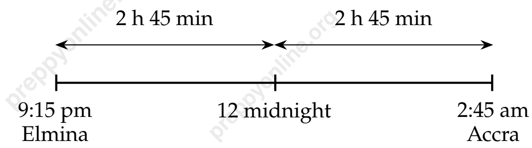
From the sequence above, we see that after the 10th leap, the frog will be 31 metres from the starting position.

Using this method, can you find the frog's distance from the starting position after leaping 101 times?

29. Answer: C

To calculate the total time the journey took, we may break the time taken into two parts, calculate the length of each part, and add them back together.

Breaking the time taken into the part before 12 midnight and the part after 12 midnight, we get the following sketch.



To find the length of the first time period, we need to find how long it is from 9:15 pm to 12 midnight. This is a time period of 2 hours 45 minutes.

To find the length of the second time period, we need to find how long it is from 12 midnight to 2:45 am. This is also a time period of 2 hours 45 minutes.

Adding the two time periods together, we get

$$\begin{aligned} 2\text{ h } 45\text{ min} + 2\text{ h } 45\text{ min} &= (2+2)\text{ h} + (45+45)\text{ min} \\ &= 4\text{ h} + 90\text{ min} \end{aligned}$$

Since 1 hour equals 60 minutes, 90 minutes equals 1 hour 30 minutes. Making this substitution gives the total time as

$$\begin{aligned} 4\text{ h} + 90\text{ min} &= 4\text{ h} + 1\text{ h } 30\text{ min} \\ &= 5\text{ h } 30\text{ min} \end{aligned}$$

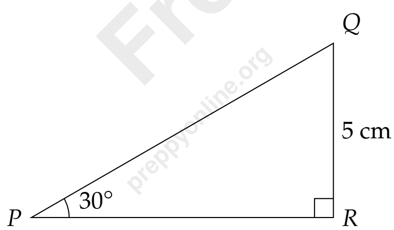
30. Answer: B

To simplify the expression, we expand brackets, group like terms, and simplify:

$$\begin{aligned} 3x - 2(3+2x) + x(2x+4) &= 3x - 6 - 4x + 2x^2 + 4x \\ &= 2x^2 + 3x - 4x + 4x - 6 \\ &= 2x^2 + 3x - 6 \end{aligned}$$

31. Answer: D

To find the length of PQ, we can try to establish a relationship between it and the 30° angle.



Using SOHCAHTOA (Section 5.15), we have

$$\sin 30^\circ = \frac{5\text{ cm}}{|PQ|}$$

Since $\sin 30^\circ = \frac{1}{2}$, we have

$$\begin{aligned} \frac{1}{2} &= \frac{5\text{ cm}}{|PQ|} \\ |PQ| &= 2(5)\text{ cm} \\ |PQ| &= 10\text{ cm} \end{aligned}$$

32. Answer: A

We shall consider two ways of solving the problem.

In the first one, we list all the possible outcomes of tossing a coin and rolling a die and check which of the outcomes satisfy the condition of Tails + odd number.

In the second one, we use the idea of independence of events to calculate the probability.

Method 1

We can represent our sample space, the set of all possible outcomes of the experiment, with the table below.

	1	2	3	4	5	6
H	H,1	H,2	H,3	H,4	H,5	H,6
T	T,1	T,2	T,3	T,4	T,5	T,6

The numbers represent the outcomes of the die roll, while the letters H and T represent Heads and Tails respectively from the coin.

Each of the 12 little squares represents a combined outcome of a coin toss and a die roll.

To get the probability of obtaining a tail and an odd number, we check which of the 12 little squares satisfy that condition and find what fraction they are of the total number of squares.

Since only the 3 shaded squares ((T,1), (T,3), and (T,5)) satisfy that condition, the probability of obtaining a tail and an odd number is

$$\frac{3}{12} = \frac{1}{4}$$

Method 2

Since the event of getting Heads or Tails on the coin does not affect what we get on rolling the die, the event of getting Tails on the coin and an odd number on the die are independent. Hence, the probability of the intersection of these events is the product of the probabilities of the individual events. That is,

$$P(\text{Tails \& odd number}) = P(\text{Tails}) \times P(\text{odd number})$$

But since we are dealing with a fair coin,

$$P(\text{Tails}) = P(\text{Heads}) = \frac{1}{2}$$

Also, since there are as many odd numbers as even numbers on a fair die,

$$P(\text{odd number}) = P(\text{even number}) = \frac{1}{2}$$

Hence,

$$P(\text{Tails \& odd number}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

33. Answer: B

If the gradient of a straight line is zero, then the line horizontal.

We can deduce this from the formula for the gradient of a line,

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x}$$

From this formula, a gradient of zero means that the change in y is zero. That means that regardless of what x is, y remains constant, and that is precisely the description of a horizontal line.

34. Answer: C

The most suitable method for collecting the data is to use a questionnaire.

Since Madam Nancy wants to know students' opinions (which teacher they like best), the most direct and reliable way to collect this kind of data is to ask the students themselves, which is done using a questionnaire.

The other options don't fit the objective she wants to achieve.

Experiment: used for testing cause and effect, not opinions

Database: stores existing data, doesn't collect new opinions

Observation: "liking" can't be accurately observed without asking

35. Answer: A

The total number of apples Mansah packed was 1,800.

If each box contained 120 apples, the number of boxes that were fully packed must have been

$$\frac{1800}{120} = 150$$

36. Answer: D

If a customer bought three items at the prices GH¢ 72.00, GH¢ 1,105.00, and GH¢ 216.00 and received a change of GH¢ 107.00, the amount he initially gave the shopkeeper must have been the sum of these amounts:

$$\text{GH¢ } (72 + 1,105 + 216 + 107) = \text{GH¢ } 1,500.00$$

37. Answer: A

Number on die	1	2	3	4	5	6
Frequency	4	3	3	2	3	5

The mode (Section 5.17.2) is the number on the die that appeared most often.

From the table, this is 6, as its frequency is greater than that of any other number.

38. Answer: D

Since every time the student rolled the die, he got one of the numbers 1, 2, 3, 4, 5, or 6, we can find the total number of times he rolled the die by adding how many times he got each of those numbers.

Doing so gives

$$4 + 3 + 3 + 2 + 3 + 5 = 20$$

39. Answer: B

When any point is rotated 360° about the origin, it returns to its original position.

Thus, the image of the given point after a 360° rotation about the origin is the same point.

40. Answer: B

In every hour, the student reads 10 pages. Hence, after t hours, the number of pages the student has read is $10t$.

If the book contains 50 pages and $10t$ of them have been read after t hours, then the number of unread pages, N, after that period is

$$N = 50 - 10t$$

In the answer options given, the term in t comes before the constant term so we have to rearrange our answer to get

$$N = 50 - 10t = -10t + 50$$

Chapter 3

2024 Paper 2

1. (a) A fair die and a fair coin are thrown together once.

(i) Write down the set of all possible outcomes.

(ii) Find the probability of obtaining a prime number and a tail.

(b) The map of a field is drawn to a scale of 1 : 100. If the width and area of the field on the map are 8 cm and 88 cm^2 respectively, find in m^2, the area of the actual field.

(c) Copy and complete the 3×3 magic square such that the sum of the numbers in each row, column and diagonal is equal to 21.

	10	3
	7	

2. (a) Given the vectors $\mathbf{p} = \begin{pmatrix} m+3 \\ 2-n \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 3m-1 \\ n-8 \end{pmatrix}$ and $\mathbf{p} = \mathbf{q}$, find the values of m and n.

(b) A man shared an amount of money between his children Baaba and William in the ratio 6 : 5. Baaba received GH¢ 1,200.00.

(i) Find the total amount shared.

(ii) William invested his share in an account at the rate of 20% simple interest per annum for 2 years. Find the total amount in his account at the end of the 2 years.

3. (a) Simplify: $3\sqrt{50} + 2\sqrt{45} - \sqrt{2} + \sqrt{5}$.

(b) A wire of length 38 cm is bent into the shape of a rectangle whose length is 7 cm more than the width. Find the area of the rectangle.

(c) If 15% of the length of a rope is 720 metres, find half of the length of the rope.

4. (a) Using a ruler and a pair of compasses only, construct $\triangle PQR$ such that angle $PQR = 90°$, $|PQ| = 5.5$ cm and $|QR| = 8$ cm.

(b) Construct a perpendicular of $\overline{PR}$ from Q.

(c) Locate M, the intersection of the perpendicular and $\overline{PR}$.

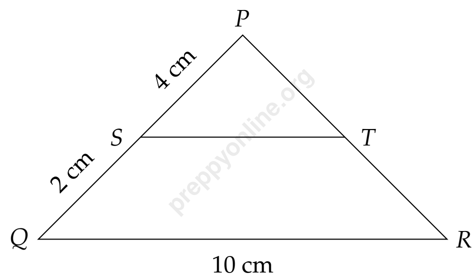
(d) Measure:

(i) $|MR|$;

(ii) $|QM|$.

(e) Calculate, correct to the nearest whole number, the area of triangle QMR.

5. (a)



NOT DRAWN TO SCALE

In the diagram, $\triangle PQR$ is an enlargement of $\triangle PST$. $|PS| = 4$ cm, $|QS| = 2$ cm and $|QR| = 10$ cm.

(i) Find the length of $\overline{ST}$.

(ii) If $|\overline{PQ}| = |\overline{PR}|$, find the area of $\triangle PQR$.

(b) The total area of a school compound is $900\frac{1}{2}$ m^2. The school has Administration and Classroom block, Library, School Park, Roads and Walkways. The areas of the Administration and Classroom block, Library and School Park are $300\frac{1}{4}$ m^2, $200\frac{1}{2}$ m^2, and

$120\frac{1}{8}$ m^2 respectively. Find the area covered by Roads and Walkways altogether.

6. (a) Copy and complete the table for the relation $F = \frac{9}{5}C + 32$, where F and C are degrees Fahrenheit and degrees Celsius respectively.

$°C$	0	5	10	15	20	25	30
$°F$	32				68		

(b) Using a scale of 2 cm to 10 units on the vertical axis ($°F$) and 2 cm to 5 units on the horizontal axis ($°C$), draw a linear graph for the relation.

(c) Use the graph to find the temperature in degrees celsius when $F = 55$ degrees.

(d) Interpret the slope of the relation.

Chapter 4

Solutions to 2024 Paper 2

Question 1

(a) (i) When a fair die is thrown, the set of possible outcomes is

$$\{1, 2, 3, 4, 5, 6\}$$

When a fair coin is thrown, we either get Heads or Tails. We normally write H for Heads and T for tails so the set of possible outcomes is

$$\{H, T\}$$

When these two experiments are done together, the set of possible outcomes is the set of all possibilities when we take one element from each set:

$$\{ (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6) \}$$

(ii) To find the probability of obtaining a prime number and a tail, we may first find the elements in the set of possible outcomes that satisfy both conditions and divide by the total number of possible outcomes.

The outcomes that include both a prime number and a tail are

$$\{(T, 2), (T, 3), (T, 5)\}$$

Since there are 3 of them and there are a total of 12 possible outcomes, the probability of obtaining a prime number and a tail is

$$\frac{3}{12} = \frac{1}{4}$$

(b) A scale of 1 : 100 means 1 unit on the map corresponds to 100 units on the actual field. Hence, 1 cm measured on the map represents 100 cm on the actual field.

What does an area of 1 cm^2 on the map represent on the actual field?

$$1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^2$$

However, since 1 cm on the map corresponds to 100 cm on the actual field, 1 cm^2 on the map corresponds to

$$100 \text{ cm} \times 100 \text{ cm} = 10,000 \text{ cm}^2 \text{ on the actual field}$$

Hence, if the area of the field on the map is 88 cm^2, the area of the actual field is

$$88 \times 10,000 \text{ cm}^2 = 880,000 \text{ cm}^2$$

To convert the area of the field from cm^2 to m^2, we use the fact that 1 m = 100 cm. Hence,

$$\begin{aligned} 1 \text{ m}^2 &= 1 \text{ m} \times 1 \text{ m} \\ &= 100 \text{ cm} \times 100 \text{ cm} \\ &= 10,000 \text{ cm}^2 \end{aligned}$$

If 1 m^2 equals 10,000 cm^2, then the field's area of 880,000 cm^2 equals

$$\frac{880,000}{10,000} = 88 \text{ m}^2$$

Be careful not to assume that the field is rectangular. This assumption is unnecessary and not supported by the information given. Although it leads to the correct answer in this case, that is only coincidental.

(c) The rules for completing the magic square are as follows:

- Each row must sum to 21
- Each column must sum to 21
- Each diagonal must sum to 21

Following these rules, we may start completing the magic square in a number of ways.

For example, let's look at row 1, shaded below.

If the three numbers in row 1 must add up to 21, what is the missing number?

The missing number must be $21 - 10 - 3 = 8$.

	10	3
	7	

$\longrightarrow$

8	10	3
	7	

Let's look at column 2, shaded below.

If the three numbers in column 2 must add up to 21, what is the missing number?

The missing number must be $21 - 10 - 7 = 4$.

8	10	3
	7	

$\longrightarrow$

8	10	3
	7	
	4	

Now let's look at the diagonal shaded below.

If the three numbers in the diagonal must add up to 21, what is the missing number?

The missing number must be $21 - 3 - 7 = 11$.

8	10	3
	7	
	4	

$\longrightarrow$

8	10	3
	7	
11	4	

Continuing in this way, we can fill all the empty squares as follows:

8	10	3
2	7	12
11	4	6

Question 2

(a) We are given the vectors $\mathbf{p} = \begin{pmatrix} m+3 \\ 2-n \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 3m-1 \\ n-8 \end{pmatrix}$.

If $\mathbf{p} = \mathbf{q}$, then

$$\begin{pmatrix} m+3 \\ 2-n \end{pmatrix} = \begin{pmatrix} 3m-1 \\ n-8 \end{pmatrix}$$

If two vectors are equal, their corresponding components must also be equal. That means,

$$m + 3 = 3m - 1 \quad \text{and} \quad 2 - n = n - 8$$

If $m + 3 = 3m - 1$, then we can group like terms and simplify to get

$$\begin{aligned} m + 3 &= 3m - 1 \\ m - 3m &= -1 - 3 \\ -2m &= -4 \\ \frac{-2m}{-2} &= \frac{-4}{-2} \\ m &= 2 \end{aligned}$$

Similarly, if $2 - n = n - 8$, we can group like terms and simplify to get

$$\begin{aligned} 2 - n &= n - 8 \\ -n - n &= -8 - 2 \\ -2n &= -10 \\ \frac{-2n}{-2} &= \frac{-10}{-2} \\ n &= 5 \end{aligned}$$

Thus, if vectors $\mathbf{p}$ and $\mathbf{q}$ are equal, the values of m and n are 2 and 5, respectively.

It is a good practice to crosscheck our answers. For example, in this question, we may substitute the values $m = 2$ and $n = 5$ into the expressions for vectors $\mathbf{p}$ and $\mathbf{q}$ to check if our answers are correct.

Substituting $m = 2$ and $n = 5$ into the expression for $\mathbf{p}$, we get

$$\mathbf{p} = \begin{pmatrix} m+3 \\ 2-n \end{pmatrix} = \begin{pmatrix} 2+3 \\ 2-5 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Doing the same substitutions for $\mathbf{q}$, we get

$$\mathbf{q} = \begin{pmatrix} 3m-1 \\ n-8 \end{pmatrix} = \begin{pmatrix} 3(2)-1 \\ 5-8 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Since we got the same result for $\mathbf{p}$ and $\mathbf{q}$, we can be sure that our answers are correct.

(b) (i) Since the man shared the money between Baaba and William in the ratio 6 : 5, the fraction of the money that Baaba received was

$$\frac{6}{6+5} = \frac{6}{11}$$

If $\frac{6}{11}$ of the total amount shared was GH¢1,200.00, what was the total amount shared?

Letting A be the total amount shared, we have the equation

$$\frac{6}{11}A = \text{GH¢ } 1,200$$

We may solve this equation to get the total amount shared, A:

$$\begin{aligned} \frac{6}{11}A &= \text{GH¢ } 1,200 \\ 6A &= \text{GH¢ } 1,200 \times 11 \\ A &= \text{GH¢ } \frac{1,200 \times 11}{6} \\ A &= \text{GH¢ } 2,200.00 \end{aligned}$$

(ii) If the total amount shared was GH¢ 2,200.00 and Baaba received GH¢ 1,200.00 of it, then William's share was

$$\text{GH¢ } 2,200 - \text{GH¢ } 1,200 = \text{GH¢ } 1,000.00$$

We are told that William invested the money he received at a rate of 20% simple interest per annum for 2 years.

To find the amount in his account after the 2 years of investing, we may find the simple interest that accrued on the principal and add it to the amount he invested.

Substituting the given values into the formula for simple interest, we get

$$\begin{aligned} \text{simple interest} &= \text{principal} \times \text{rate} \times \text{time} \\ &= \text{GH¢ } 1{,}000 \times 20\% \times 2 \\ &= \text{GH¢ } 1{,}000 \times \frac{20}{100} \times 2 \\ &= \text{GH¢ } 400.00 \end{aligned}$$

Adding the simple interest to the amount he invested, we get the amount in his account as

$$\text{GH¢ } 1{,}000 + \text{GH¢ } 400 = \text{GH¢ } 1,400.00$$

Question 3

(a) We apply the rules of surds to the given expression and simplify:

$$\begin{aligned} & 3\sqrt{50} + 2\sqrt{45} - \sqrt{2} + \sqrt{5} \\ &= 3\sqrt{25 \times 2} + 2\sqrt{9 \times 5} - \sqrt{2} + \sqrt{5} \\ &= 3\sqrt{25}\sqrt{2} + 2\sqrt{9}\sqrt{5} - \sqrt{2} + \sqrt{5} \\ &= 3(5)\sqrt{2} + 2(3)\sqrt{5} - \sqrt{2} + \sqrt{5} \\ &= 15\sqrt{2} + 6\sqrt{5} - \sqrt{2} + \sqrt{5} \\ &= 15\sqrt{2} - \sqrt{2} + 6\sqrt{5} + \sqrt{5} \\ &= 14\sqrt{2} + 7\sqrt{5} \end{aligned}$$

(b) Let the width of the rectangle be w cm.

Since the length of the rectangle is 7 cm longer than the width, the length is $(w+7)$ cm.

Since the rectangle was created from a wire of length 38 cm, its perimeter is 38 cm.

But the perimeter of a rectangle is given by the formula

$$\text{perimeter} = 2(\text{length} + \text{width})$$

Substituting the values into the formula and solving, we get

$$\begin{aligned} \text{perimeter} &= 2(\text{length} + \text{width}) \\ 38 &= 2((w+7)+w) \\ 38 &= 2(w+7+w) \\ 38 &= 2(2w+7) \\ \frac{38}{2} &= \frac{2(2w+7)}{2} \\ 19 &= 2w+7 \\ 19-7 &= 2w \\ 12 &= 2w \\ \frac{12}{2} &= \frac{2w}{2} \\ 6 &= w \\ w &= 6 \text{ cm} \end{aligned}$$

If the width is 6 cm, then the length is

$$\text{width} + 7 \text{ cm} = 6 \text{ cm} + 7 \text{ cm} = 13 \text{ cm}$$

If the length of the rectangle is 13 cm and its width is 6 cm, then its area is

$$\text{area} = \text{length} \times \text{width} = 13 \text{ cm} \times 6 \text{ cm} = 78 \text{ cm}^2$$

(c) If 15% of the length of a rope is 720 metres, find half of the length of the rope.

Half of the length of the rope is the same as 50% of the length of the rope as

$$\frac{1}{2} = \frac{50}{100} = 50\%$$

Hence, our question becomes: If 15% of the length of a rope is 720 metres, what is 50% of the length of the rope?

We can solve this by finding 1% of the length of the rope.

If 15% of the length of the rope is 720 metres, then 1% of the length of the rope is

$$\frac{720 \text{ metres}}{15}$$

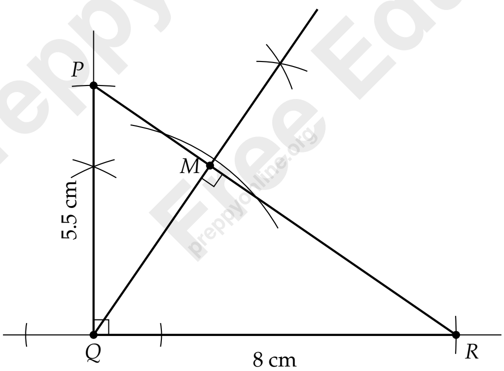
If 1% of the length of the rope is the expression above, then 50% of the length of the rope is

$$\frac{720 \text{ metres}}{15} \times 50$$

Simplifying this expression gives us the answer as 2,400 metres.

Question 4

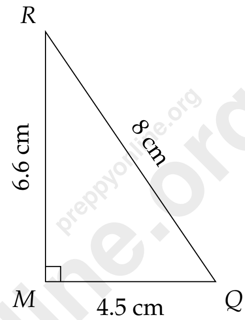
(a–c) The diagram below has been constructed to scale. Although the lengths are reduced to fit on the page, they remain proportional to the actual lengths. The scale used is 1 cm in the actual construction to 0.6 cm in the diagram. Thus, all lengths in the diagram are 60% of the actual lengths.



(d) (i) From the construction, we can measure length $|MR|$ as 6.6 cm, correct to one decimal place.

(ii) We can also measure $|QM|$ as 4.5 cm, correct to one decimal place.

(e) Triangle QMR is sketched below. The 4.5 cm and 6.6 cm lengths are what we got from the construction.



The formula for the area of a triangle (Section 5.7.2) is

$$\text{area} = \frac{1}{2} \times \text{base} \times \text{height}$$

In our case, we may take the base of the triangle as $|MQ|$ and the height as $|MR|$.

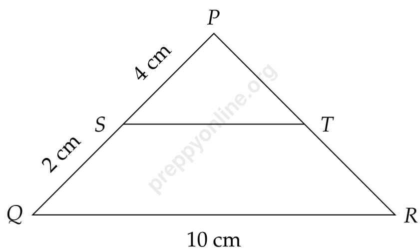
Doing so and substituting the values into the formula, we get

$$\begin{aligned} \text{area} &= \frac{1}{2} \times 4.5 \text{ cm} \times 6.6 \text{ cm} \\ &= 14.85 \text{ cm}^2 \end{aligned}$$

This is equal to 15 cm^2 to the nearest whole number.

Question 5

(a) (i)



Since triangle PQR is an enlargement of triangle PST, pairs of corresponding sides of the two triangles are in the same ratio.

That means

$$\frac{|PS|}{|PQ|} = \frac{|ST|}{|QR|}$$

From the figure, we have $|PS| = 4$ cm, $|PQ| = 6$ cm, and $|QR| = 10$ cm. Substituting these values into the expression above gives

$$\frac{4}{6} = \frac{|ST|}{10}$$

Multiplying both sides by 10, we get

$$\begin{aligned} |ST| &= \frac{4}{6} \times 10 \\ &= \frac{40}{6} \text{ cm} \\ &= \frac{20}{3} \text{ cm} \end{aligned}$$

$\frac{20}{3}$ cm is equivalent to $6\frac{2}{3}$ cm or 6.67 cm to two decimal places. Any of those answers is acceptable.

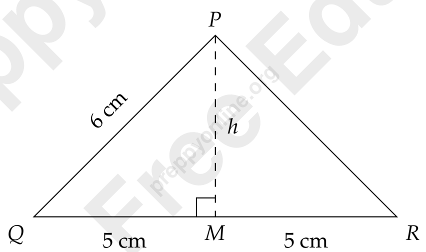
(ii) To find the area of triangle PQR, we may use the formula for the area of a triangle:

$$\text{area} = \frac{1}{2} \times \text{base} \times \text{height}$$

If we take $|QR|$ as the base, then we need to calculate the perpendicular height from P to the base QR.

Since $|\overline{PQ}| = |\overline{PR}|$, triangle PQR is isosceles. That means if we drop a perpendicular from P to line segment QR, the perpendicular will bisect line segment QR.

Hence, we have the sketch below:



From the sketch, triangle MPQ is a right-angled triangle with hypotenuse PQ so we may apply Pythagoras' theorem to write

$$|PQ|^2 = |QM|^2 + |MP|^2$$

Substituting $|QM| = 5$ cm, $|PQ| = 6$ cm, and $|MP| = h$ into the expression, we get

$$\begin{aligned} |PQ|^2 &= |QM|^2 + |MP|^2 \\ 6^2 &= 5^2 + h^2 \\ 36 &= 25 + h^2 \\ 36 - 25 &= h^2 \\ 11 &= h^2 \\ \sqrt{11} &= \sqrt{h^2} \\ \sqrt{11} &= h \\ h &= \sqrt{11} \text{ cm} \end{aligned}$$

Thus, we now know the perpendicular height and the base. Hence, we may compute the area as

$$\begin{aligned} \text{area} &= \frac{1}{2} \times 10 \text{ cm} \times \sqrt{11} \text{ cm} \\ &= 5\sqrt{11} \text{ cm}^2 \end{aligned}$$

(b) The total area of a school compound is $900\frac{1}{2}$ m^2. The school has Administration and Classroom block, Library, School Park, Roads and Walkways. The areas of the Administration and Classroom block, Library and School Park are $300\frac{1}{4}$ m^2, $200\frac{1}{2}$ m^2, and $120\frac{1}{8}$ m^2 respectively. Find the area covered by Roads and Walkways altogether.

The total are of the school compound is the sum of the following areas:

- Administration and Classroom block: $300\frac{1}{4}$ m^2
- Library: $200\frac{1}{2}$ m^2
- School Park: $120\frac{1}{8}$ m^2
- Roads and Walkways: not given

If we add the areas of the four regions above, we should get the total area of the school compound.

Hence, to find the total area covered by Roads and Walkways, we may add up the areas of the other regions and subtract the total from the total area of the school compound.

Adding up the areas of the other regions, we get

$$
\begin{aligned}
&300\tfrac{1}{4} + 200\tfrac{1}{2} + 120\tfrac{1}{8}\\
&= (300 + 200 + 120) + \left(\frac{1}{4} + \frac{1}{2} + \frac{1}{8}\right)\\
&= 620 + \frac{1(2) + 1(4) + 1(1)}{8}\\
&= 620 + \frac{2 + 4 + 1}{8}\\
&= 620 + \frac{7}{8}\\
&= 620\tfrac{7}{8}
\end{aligned}
$$

Subtracting $620\frac{7}{8}$ from the total area of the school compound, we get

$$
\begin{aligned}
900\tfrac{1}{2} - 620\tfrac{7}{8} &= \left(900 + \frac{1}{2}\right) - \left(620 + \frac{7}{8}\right)\\
&= 900 + \frac{1}{2} - 620 - \frac{7}{8}\\
&= 900 - 620 + \frac{1}{2} - \frac{7}{8}\\
&= 280 + \frac{1(4) - 7(1)}{8}\\
&= 280 + \frac{4 - 7}{8}\\
&= 280 + -\frac{3}{8}\\
&= 280 - \frac{3}{8}\\
&= 279 + 1 - \frac{3}{8}\\
&= 279 + \frac{8 - 3}{8}\\
&= 279 + \frac{5}{8}\\
&= 279\tfrac{5}{8}
\end{aligned}
$$

Hence, the area covered by Roads and Walkways is $279\frac{5}{8}$ m^2.

Question 6

(a) Given the relation $F = \frac{9}{5}C + 32$, when $C = 5$,

$$F = \frac{9}{5}(5) + 32 = 9 + 32 = 41$$

When $C = 10$, we have

$$F = \frac{9}{5}(10) + 32 = 18 + 32 = 50$$

When $C = 15$, we have

$$F = \frac{9}{5}(15) + 32 = 27 + 32 = 59$$

When $C = 25$, we have

$$F = \frac{9}{5}(25) + 32 = 45 + 32 = 77$$

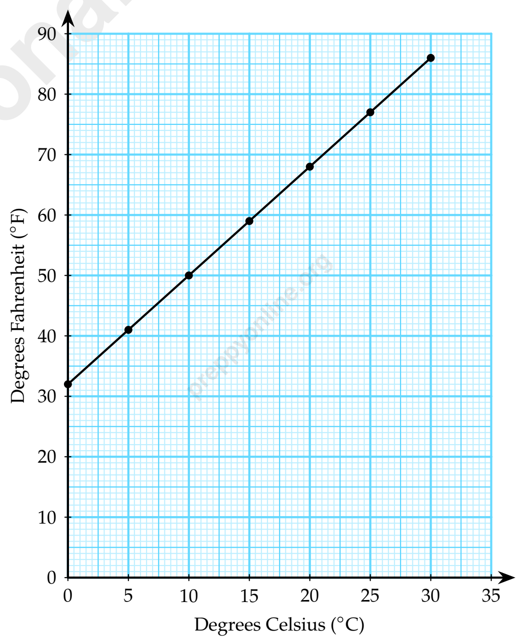
When $C = 30$, we have

$$F = \frac{9}{5}(30) + 32 = 54 + 32 = 86$$

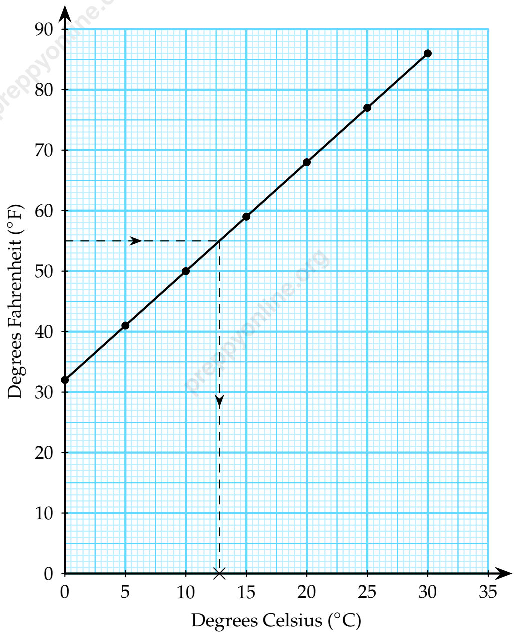
Using these values, we may complete the table as follows:

°C	0	5	10	15	20	25	30
°F	32	41	50	59	68	77	86

(b) Using a scale of 2 cm to 10 units on the vertical axis (°F) and 2 cm to 5 units on the horizontal axis (°C), we obtain the following graph for the relation.



(c) To use the graph to find the temperature in degrees celsius when $F = 55$ degrees, we draw a horizontal line from the 55°F mark to touch the line for the relation and project this line vertically downwards to see where it lands on the °C axis.



Doing so, we see that the Celsius value that corresponds to 55°F is about halfway between 12.5°C and 13°C. So we estimate it as

$$\frac{12.5 + 13.0}{2} = 12.75°\text{C}$$

(d) From the equation of the relation, $F = \frac{9}{5}C + 32$, the slope is $\frac{9}{5}$.

This tells us that a temperature increase of 1°C corresponds to a temperature increase of $\frac{9}{5}$°F

Chapter 5

Quick Revision Notes

5.1 Sets

A set is a well-defined collection of unique objects.

An example of a set is the set of counting numbers less than 5: $\{1, 2, 3, 4\}$.

Another example is the set of colors in a rainbow: $\{\text{red}, \text{orange}, \text{yellow}, \text{green}, \text{blue}, \text{indigo}, \text{violet}\}$.

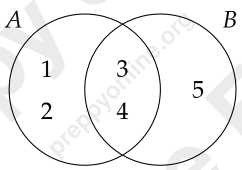
Note that the elements of a set must be unique. That means we can't have repeated elements in a set. For example, $\{1, 2, 2, 3\}$ is not set because 2 is repeated.

Sets are often denoted by capital letters, e.g., $E = \{2, 4, 6, 8\}$.

5.1.1 Venn diagrams

Venn diagrams are a useful tool for showing the relationships between sets.

If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$, we can represent the relationship between A and B with the diagram below.



Notice that because the elements 3 and 4 are in both sets, they are shown in the part of the diagram where the two circles that represent the two sets overlap.

5.1.2 Types of sets

Finite sets

A finite set is a set with a countable number of elements. That means its number of elements can be represented by a non-negative integer such as 0, 1, 2, 3, and so on.

An example of a finite set is the set of vowels in the English alphabet: $\{a, e, i, o, u\}$.

Infinite sets

An infinite set is a set with an unlimited number of elements. That means its number of elements cannot be represented by a natural number.

An example of an infinite set is the set of whole numbers, $\{0, 1, 2, 3, 4, 5, \dots\}$. This set is infinite because there is no such thing as the largest whole number. Hence, the set is endless.

The empty set

The empty set is the set with no elements. That means its number of elements is 0.

They empty set is denoted by empty curly braces, $\{\}$. It is also denoted by $\varnothing$.

Singletons

A singleton is a set with exactly one element. That means its number of elements is 1.

Examples of singleton sets are the following

$$\{8\} \quad \{\text{A}\} \quad \{0\} \quad \{\text{orange}\}$$

Is the set $\{\{1, 2, 3\}\}$ a singleton?

Yes, it is. It has only one element, the set $\{1, 2, 3\}$.

5.1.3 Subsets

Given a set A, a subset of A is a set whose elements are also elements of A.

For instance, the set $\{3, 4\}$ is a subset of $\{1, 2, 3, 4, 5\}$ because each of its elements—3 and 4—is an element of the set $\{1, 2, 3, 4, 5\}$.

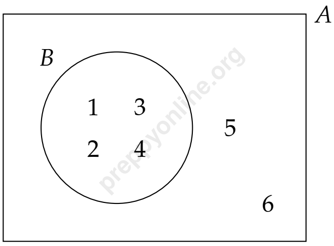
Is $\{1, 2, 3, 4, 5\}$ a subset of $\{1, 2, 3, 4, 5\}$?

Yes, it is. Using the definition above, we have to check if every element of $\{1, 2, 3, 4, 5\}$ is an element of $\{1, 2, 3, 4, 5\}$. As this is obviously the case, $\{1, 2, 3, 4, 5\}$ is a subset of itself.

By this reasoning, every set is a subset of itself.

We write $A \subseteq B$ to denote that A is a subset of B.

The diagram below shows set A, represented by the rectangle, and set B represented by the circle. From the diagram, set B is a subset of set A because every element in B is also in A.



The elements of B are 1, 2, 3, 4, while the elements of A are 1, 2, 3, 4, 5, 6.

Proper subsets

Sometimes we only want subsets of a set that are not the set itself. These are called proper subsets. Hence, though every set is a subset of itself, it is not a proper subset of itself.

$\{3,4,5\}$ is a proper subset of $\{1,2,3,4,5\}$ but $\{1,2,3,4,5\}$ is not a proper subset of $\{1,2,3,4,5\}$.

5.1.4 The number of subsets of a set

The number of subsets of a set with n elements is 2^n.

For example, the set $\{1,2,3\}$ has $2^3 = 8$ subsets because it has 3 elements. The 8 subsets are the following:

$$\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$$

Notice that the list of subsets includes the empty set, $\{\}$, and the set itself $\{1,2,3\}$. The empty set is a subset of every set. Every set is also a subset of itself.

Since $\{1,2,3,4,5\}$ has 5 elements, it has $2^5 = 32$ subsets. Can you list them?

5.1.5 Comparing sets

Equal sets

Two sets are equal if they have the same elements. For example, $\{1,2,3\}$ and $\{3,2,1\}$ are equal because they have the same elements. This means the order in which the elements of a set are arranged does not matter. Once two sets have the same elements, they are equal.

Equivalent sets

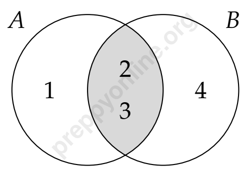
Two sets are equivalent if they have the same number of elements. For example, $\{1,2,3\}$ and $\{4,5,6\}$ are equivalent because they have the same number of elements, 3.

Disjoint sets

Two sets are called disjoint if they have no elements in common. For example, $A = \{1,2\}$ and $B = \{3,4,5\}$ are disjoint because they have no elements in common.

Intersecting sets

If two sets are not disjoint, then they are intersecting sets. Intersecting sets are sets that have elements in common. For example, $A = \{1,2,3\}$ and $B = \{2,3,4\}$ are intersecting sets because they have 2 and 3 in common. Their intersection is shaded in the Venn diagram below.



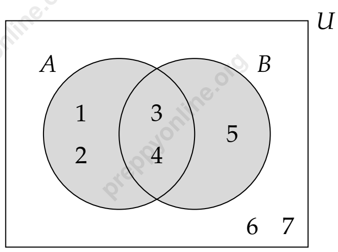
5.1.6 Operations on sets

Union

The union of sets A and B, denoted $A \cup B$, is the set that contains all the elements of both sets.

For example, the union of $A = \{1,2,3\}$ and $B = \{2,3,4\}$ is $A \cup B = \{1,2,3,4\}$.

Let A and B be subsets of the universal set $U = \{1,2,3,4,5,6,7\}$. If $A = \{1,2,3,4\}$ and $B = \{3,4,5\}$, then $A \cup B = \{1,2,3,4,5\}$ and the relationships between the sets can be represented by the Venn diagram below. The shaded region represents the union.

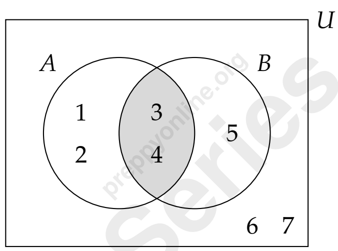


Intersection

The intersection of sets A and B, denoted $A \cap B$, is the set that contains all the elements that are in both sets.

For example, the intersection of $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$ is $A \cap B = \{2, 3\}$.

Let A and B be subsets of the universal set $U = \{1, 2, 3, 4, 5, 6, 7\}$. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$, then $A \cap B = \{3, 4\}$ and the relationships between the sets can be represented by the Venn diagram below. The shaded region represents the intersection.

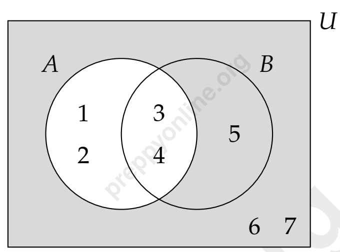


Complement

The complement of a set is the set that contains all the elements that are not in the set. For example, the complement of $A = \{1, 2, 3\}$ in $\{1, 2, 3, 4, 5\}$ is $A' = \{4, 5\}$. The complement of A is denoted A'.

Whenever we talk of complements, we must do so in reference to a larger set. For example, the complement of $\{1, 2, 3\}$ in $\{1, 2, 3, 4, 5\}$ is $\{4, 5\}$. But the complement of $\{1, 2, 3\}$ in $\{1, 2, 3, 4, 5, 6\}$ is $\{4, 5, 6\}$.

Let A and B be subsets of the universal set $U = \{1, 2, 3, 4, 5, 6, 7\}$. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$, then $A' = \{5, 6, 7\}$ and the relationships between the sets can be represented by the Venn diagram below. The shaded region represents A', the complement of A.



5.2 Real numbers

The set of real numbers is, roughly speaking, the set of all numbers that can be represented on a number line. It comprises the set of rational numbers (positive and negative integers, zero, and fractions) and the set of irrational numbers (non-repeating decimals like π and $\sqrt{2}$).

5.2.1 Properties of arithmetic operations on real numbers

The commutative property

An operation is said to be commutative if the order of the operands does not change the result.

For example, addition is commutative because the order in which numbers are added in a sum does not change the answer.

For example, $2 + 3 = 3 + 2$.

Similarly, $2 + 3 + 4 = 4 + 2 + 3 = 3 + 2 + 4$.

Subtraction is not commutative. This means that when subtracting numbers, order matters. Changing the order of the numbers will give a different answer.

For example, $2 - 3 \neq 3 - 2$.

Multiplication is commutative. Again, this means that when multiplying numbers, we may do so in any order.

For example, $2 \times 3 = 3 \times 2$.

Similarly, $2 \times 3 \times 4 = 4 \times 2 \times 3 = 3 \times 2 \times 4$.

Division is not commutative. Order is important when dividing. $4 \div 2 \neq 2 \div 4$.

The associative property

This property is about the grouping of numbers when doing arithmetic with three or more numbers. If changing the grouping of the operands does not change the result, the operation is associative.

For example, when adding three numbers, it does not matter how we group the numbers with parentheses:

$$(1 + 2) + 3 = 1 + (2 + 3)$$

This is because addition of numbers is associative.

Multiplication is also associative. For example, when multiplying three numbers, it does not matter how we group them:

$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

Subtraction and division are, however, not associative. Changing the grouping of the numbers changes the result. For example,

$$(1 - 2) - 3 = -4 \quad \text{but} \quad 1 - (2 - 3) = 2$$

The distributive property

The distributive property of multiplication over addition is the property that when multiplying a sum by a number, we can first take the sum and multiply by the number or multiply the number by each of the addends and then take the sum of these new products.

For example,

$$2 \times (3 + 4) = 2 \times 7 \quad \text{and also} \quad 2 \times (3 + 4) = 2 \times 3 + 2 \times 4$$

5.2.2 Prime factorization

A prime number is a natural number greater than 1 whose only factors are 1 and itself.

Prime factorization is the process of breaking up a composite number into a product of prime numbers.

Every integer greater than 1 is either prime or the product of a collection of primes.

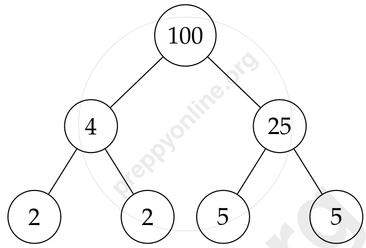
For example,

- 5 is prime.
- 15 is composite. It is the product of 3 and 5.
- 100 is composite. It is the product of the primes 2, 2, 5, and 5.
- 13 is prime.

To break an integer into a product of primes, we may first break it up into a product of any two integers and then break up those integers into a product of integers until we get a product of primes. This process can be illustrated with a factor tree.

For example, $100 = 4 \times 25$. Both 4 and 25 are not prime but can be written as products of primes: $4 = 2 \times 2$ and $25 = 5 \times 5$. Thus, $100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$.

This is what the factor tree below shows.



5.2.3 Standard form

To write a number in standard form, we need to write it in the form $a \times 10^n$ where $1 \leqslant a < 10$ and n is an integer.

For example, to write 0.00459 in standard form, we need to keep moving the decimal point to the right until it is after the first non-zero number. Doing that while keeping track of the number of rimes we moved, we get

$$0.00459 = 4.59 \times 10^{-3}$$

5.3 Percentages

A percentage is a fraction of 100.

For example,

$$50\% = \frac{50}{100}$$

To express a number as a percentage, we multiply by 100%.

For example, $\frac{1}{4}$ can be expressed as a percentage as

$$\frac{1}{4} \times 100\% = 25\%$$

5.4 Simple interest

The formula for simple interest is

$$\text{simple interest} = \text{principal} \times \text{rate} \times \text{time}$$

The principal is the amount borrowed or invested.

The rate is how much the principal increases per given period. It can be stated per annum (same as per year), per month, per week, etc.

The time is how long the amount is borrowed or invested.

For example, to find the simple interest on GH¢ 2,000.00 invested at 10% simple interest per annum for a period of 9 months, we extract the relevant information.

The principal is GH¢ 2,000.00.

The rate is 10% per annum or per year.

The time is 9 months. However, because the rate is given in terms of years, we would like to convert the time also to years.

Since there are 12 months in a year, 9 months is equivalent to $\frac{9}{12}$ years.

With these three values, the simple interest may be computed as

$$\begin{aligned} \text{simple interest} &= \text{GH¢ } 2000 \times 10\% \times \frac{9}{12} \\ &= 2000 \times \frac{10}{100} \times \frac{9}{12} \\ &= \text{GH¢ } 150.00 \end{aligned}$$

5.5 Ratio and proportion

5.5.1 Ratios

A ratio is a way of expressing the relationship between two numbers.

For example, if there are 4 mangoes and 6 oranges in a basket, the ratio of mangoes to oranges is 4 : 6.

This ratio may also be written as $\frac{4}{6}$. Hence, ratios and fractions can be used to represent the same information.

Just as the fraction $\frac{4}{6}$ is equivalent to $\frac{2}{3}$, the ratio 4 : 6 is equivalent to 2 : 3.

Ratios tell us how a number is divided and the relative sizes of the portions.

For example, if the ratio of red pens to blue pens in a box is 2 : 3 and there is a total of 15 pens in the box, how many are red?

The ratio 2 : 3 tells us that for every 2 red pens there are, there are 3 blue pens.

Thus, if we see 4 red pens, we should see 6 blue pens; if we see 6 red pens, there should be 9 blue pens, and so on.

The fraction of pens in the box that are red is

$$\frac{2}{2+3} = \frac{2}{5}$$

Hence, the number of red pens is

$$\frac{2}{5} \times 15 = 6$$

5.5.2 Proportions

We get a proportion when we set two ratios equal to each other.

For example, if the ratio of boys to girls in a class is 3 : 4, and there are 15 boys in the class, how many girls are there?

This information can be presented as

$$\begin{aligned} \text{boys} &: \text{girls} \\ 3 &: 4 \\ 15 &: x \end{aligned}$$

Since the three ratios above are equal, we can write

$$3 : 4 = 15 : x$$

When we do that, we get a proportion.

Since ratios can be written as fractions, we have

$$\frac{3}{4} = \frac{15}{x}$$

Cross-multiplying and solving for x gives

$$\begin{aligned} 3x &= 15(4) \\ x &= \frac{15(4)}{3} \\ x &= 20 \end{aligned}$$

5.5.3 Direct proportion

When the relationship between two quantities is such that as one increases, the other increases and when one decreases, the other decreases, there is direct relationship between them.

For example, if 10 cows eat 20 bags of food, how many bags of food will 20 cows eat?

20 cows will eat more food than 10 cows. In fact, they will eat twice as much food.

The information can be written as

$$\begin{aligned} 10 \text{ cows} &\rightarrow 20 \text{ bags} \\ 20 \text{ cows} &\rightarrow x \text{ bags} \end{aligned}$$

Since there is a direct relationship between the number of cows and the number of bags, we can write these equivalent fractions:

$$\frac{10}{20} = \frac{20}{x}$$

Cross-multiplying and solving for x gives

$$\begin{aligned} 10x &= 20(20) \\ x &= \frac{20(20)}{10} \\ x &= 40 \end{aligned}$$

5.5.4 Indirect or inverse proportion

When the relationship between two quantities is such that as one increases, the other decreases and when one decreases, the other increases, there is indirect or inverse relationship between them.

For example, if 5 men use 20 hours to paint a house, how many hours will 10 men use to paint it?

10 men will use less time to paint the house than 5 men will. In fact, they will use half the amount of time 5 men will use.

The information can be written as

$$5 \text{ men} \rightarrow 20 \text{ hours}$$
$$10 \text{ men} \rightarrow x \text{ hours}$$

Because of the inverse relationship between the number of men and the amount of time they will need, we can write these equivalent fractions:

$$\frac{5}{10} = \frac{x}{20}$$

Notice that because of the inverse relationship between the number of men and the number of hours, we flipped the order of the quantities on the right-hand side of the proportion.

Solving for x in the proportion above gives $x = 10$.

5.6 Factorization of algebraic expressions

5.6.1 The difference of two squares

Expressions of the form $a^2 - b^2$, where one square is subtracted from another have may be factorized as

$$a^2 - b^2 = (a - b)(a + b)$$

This is an important factorization that has many applications in mathematics and should be memorized.

The factorization may be verified by expanding it:

$$(a - b)(a + b) = a^2 + ab - ab - b^2 = a^2 - b^2$$

Using the difference of two squares factorization, $4x^2 - 9y^2$ can be factorized as

$$4x^2 - 9y^2 = (2x)^2 - (3y)^2 = (2x - 3y)(2x + 3y)$$

To evaluate $65^2 - 35^2$, we may use the difference of two squares factorization to write

$$65^2 - 35^2 = (65 - 35)(65 + 35) = 30(100) = 3000$$

5.7 Mensuration

Mensuration is all about measurement. That is, the measurement of distances, areas, volumes, etc.

5.7.1 Perimeter

The perimeter of a plane figure is the sum of the lengths of its sides or the distance around its edge.

The circumference of a circle with radius r is given by the formula

$$\text{circumference} = 2\pi r$$

The perimeter of a rectangle is given by the formula

$$\text{perimeter} = 2 \times (l + w)$$

5.7.2 Area

The area of a circle with radius r is given by the formula

$$\text{area} = \pi r^2$$

The area of a rectangle with length l and width w is given by the formula

$$\text{area} = l \times w$$

The area of a square with side length a is given by the formula

$$\text{area} = a^2$$

The area of a triangle with base b and perpendicular height h is given by the formula

$$\text{area} = \frac{1}{2} \times b \times h$$

The area of a trapezium with parallel sides a and b and perpendicular height h is given by the formula

$$\text{area} = \frac{1}{2} \times (a + b) \times h$$

The area of a parallelogram with base b and perpendicular height h is given by the formula

$$\text{area} = b \times h$$

5.7.3 Volume

The volume of a cube with side length a is given by the formula

$$\text{volume} = a^3$$

The volume of a cylinder with radius r and height h is given by the formula

$$\text{volume} = \pi r^2 h$$

The volume of a sphere with radius r is given by the formula

$$\text{volume} = \frac{4}{3}\pi r^3$$

5.8 Rules of indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

5.9 Rules of logarithms

Product rule

$$\log_b(xy) = \log_b x + \log_b y$$

For example,

$$\log_2 64 = \log_2(8 \times 8) = \log_2 8 + \log_2 8 = 3 + 3 = 6$$

Quotient rule

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

For example,

$$\log_2\left(\frac{32}{4}\right) = \log_2 32 - \log_2 4 = 5 - 2 = 3$$

Power rule

$$\log_b(x^p) = p\log_b x$$

For example,

$$\log_3 81 = \log_3 3^4 = 4\log_3 3 = 4$$

Root rule

$$\log_b \sqrt[p]{x} = \frac{\log_b x}{p}$$

For example,

$$\begin{aligned}\log_3 \sqrt{243} &= \log_3 243^{\frac{1}{2}}\\ &= \frac{1}{2}\log_3 243\\ &= \frac{1}{2}\log_3 3^5\\ &= \frac{1}{2}\times 5\\ &= \frac{5}{2}\end{aligned}$$

5.10 Surds

A surd is an irrational number expressed as a root (such as a square root or a cube root) that cannot be simplified into a whole number or exact fraction.

$\sqrt{2}$ is a surd because it cannot be simplified into a whole number or exact fraction.

$\sqrt{4}$ is not a surd because it can be simplified into the whole number 2.

$\sqrt{\frac{25}{4}}$ is not a surd because it can be simplified into the fraction $\frac{5}{2}$.

$sqrt32$ is a surd because it cannot be simplified into a whole number or exact fraction as 32 is not a perfect square.

We will restrict ourselves to square roots but the principles for other roots are similar.

5.10.1 Rules of surds

The product rule

$$\sqrt{a}\sqrt{b} = \sqrt{ab}$$

For example,

$$\sqrt{2}\sqrt{2} = \sqrt{2\times 2} = \sqrt{4} = 2$$

Also,

$$\sqrt{5}\sqrt{2} = \sqrt{10}$$

The quotient rule

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

For example,

$$\frac{\sqrt{32}}{\sqrt{2}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4$$

5.10.2 Simplification of surds

To simplify a surd, we try to write the number under the radical as a product of a perfect square and another number.

For example, since $32 = 16 \times 2$, $\sqrt{32}$ may be written as

$$\sqrt{32} = \sqrt{16 \times 2}$$

Then, applying the rules of surds, we get

$$\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4 \times \sqrt{2} = 4\sqrt{2}$$

Similarly,

$$\sqrt{99} = \sqrt{9 \times 11} = \sqrt{9}\sqrt{11} = 3\sqrt{11}$$

5.10.3 Rationalization of the denominator

This is a technique used to eliminate surds from the denominator of a fraction. We multiply both the numerator and the denominator by a special number that the value of the fraction is not changed but the denominator becomes a rational number.

For example, $\frac{2}{\sqrt{3}}$ is the same as

$$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

When the denominator of the fraction is a sum or a difference involving a radical (for example, $a + \sqrt{b}$), we multiply by both the numerator and the denominator by the conjugate of the surd, which is the expression with the opposite sign between the two terms. That is, the conjugate of $a + \sqrt{b}$ is $a - \sqrt{b}$ and vice versa.

For example, to rationalize the denominator of $\frac{2}{2+\sqrt{3}}$, we multiply both the numerator and the denominator of the fraction by the conjugate of $2 + \sqrt{3}$:

$$\frac{2}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}$$

Using the fact that the denominator is now a difference of two squares (Section 5.6.1), we have

$$\begin{aligned}\frac{2\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)} &= \frac{2\left(2-\sqrt{3}\right)}{2^2-\left(\sqrt{3}\right)^2} \\ &= \frac{4-2\sqrt{3}}{4-3} \\ &= \frac{4-2\sqrt{3}}{1} \\ &= 4-2\sqrt{3}\end{aligned}$$

5.11 Sequences and series

Sometimes a sequence of numbers follows a pattern by which we can deduce what the other numbers in the sequence are.

For example, can you find the next number in the sequence below?

$$1, 2, 4, 7, 11, \ldots$$

If we notice the pattern below, we can guess the next number as 16.

$$1 \xrightarrow{+1} 2 \xrightarrow{+2} 4 \xrightarrow{+3} 7 \xrightarrow{+4} 11 \xrightarrow{?} \ldots$$

5.11.1 Arithmetic progressions or linear sequences

An arithmetic progression (AP) or linear sequence is a sequence of numbers in which the difference between two consecutive terms is constant. This constant difference between the terms of the sequence is called the common difference.

For example, $1, 2, 3, 4, \ldots$ is an arithmetic progression because every two consecutive terms in the sequence differ by 1. In other words, given a term of the sequence, we obtain the next term by adding 1.

$2, 4, 6, 8, \ldots$ is also an arithmetic progression because the difference between every pair of consecutive terms is constant. In particular, given a term of the sequence, the next term is obtained by adding 2.

The first term and the common difference

What is the common difference of the arithmetic progression $4, 2, 0, -2, \ldots$?

Given a term of the sequence, we must add -2 to it to get the next term. Hence, the common difference is -2.

The difference between consecutive terms of an arithmetic progression is called the common difference. It is usually denoted by d.

The first term of an arithmetic progression is usually denoted by a.

Hence, in the sequence $1, 2, 3, 4, \ldots$, the first term, $a = 1$ and the common difference, $d = 1$.

In the sequence $2, 4, 6, 8, \ldots$, the first term, $a = 2$ and the common difference, $d = 2$.

The nth term of an AP

The nth term of an arithmetic progression, denoted u_n, is given by the formula

$$u_n = a + (n - 1)d$$

For example, given the sequence $3, 5, 7, 9, \ldots$, since the first term, $a = 3$, and the common difference, $d = 2$, the 10th term, u_{10}, is given by

$$u_{10} = 3 + (10 - 1)2 = 3 + 9(2) = 3 + 18 = 21$$

Notice that we could have found the 10th term by continuing the sequence until we got to the 10th term. However, it is faster to use the formula.

5.11.2 Geometric progressions or exponential sequences

A geometric progression or exponential sequence is a sequence of numbers where successive terms are obtained by multiplying the same number, called the common ratio.

An example of a geometric progression is $1, 2, 4, 8, \ldots$. Each term in the sequence is obtained by multiplying the preceding term by the constant 2.

Another example is $81, 27, 9, 3, \ldots$. This is a geometric progression because subsequent terms are obtained by multiplying the preceding term by a constant, $\frac{1}{3}$.

The first term and the common ratio

Find the common ratio of the geometric progression $64, 32, 16, 8, \ldots$.

The common ratio is the number that we must multiply a given term by to get the next term. Given 64, the next term is 32. Since we must multiply 64 by $\frac{1}{2}$ to get 32, the common ratio is $\frac{1}{2}$.

The first term of a geometric progression is usually denoted a, while the common ratio is denoted r.

The nth term of a GP

The nth term of a geometric progression, denoted u_n, is given by the formula

$$u_n = ar^{n-1}$$

For example, since the first term of the geometric progression $1, 2, 4, 8, \ldots$ is 1 and the common ratio is 2, the 10th term of the sequence is

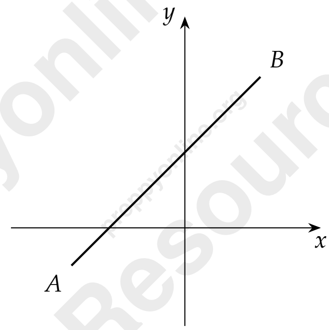
$$u_{10} = 1(2)^{10-1} = 2^9 = 512$$

5.12 Coordinate geometry

A straight line is the shortest distance between two points.

They have many applications in mathematics.

An example of a straight line is line AB below.



5.12.1 The gradient or slope of a line

Imagine climbing a mountain. The steeper the slope, the harder it is to climb; and the gentler the slope, the easier it is to climb.

The gradient or slope of a line is a measure of how steep the line is.

For example, lines A and B below have gentle slopes,

while lines C and D below have steep slopes.

To find the gradient of a line, we need to find to points (x_1, x_2) and (y_1, y_2) on the line and use the formula

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

For example, to find the gradient of the line that passes through the points $(2, 5)$ and $(-1, 7)$, we let one pair of coordinates be (x_1, y_1) and the other one be (x_2, y_2) and use the formula to get the gradient as

$$\frac{7-5}{-1-2} = \frac{2}{-3} = -\frac{2}{3}$$

5.12.2 The equation of a line

The equation of a line may be written in different forms.

The general form of the equation of a line

The general form of the equation of a line is

$$ax + by + c = 0$$

This means that equation of every line can be written in this form. We can't have terms like x^2, x^3, y^2, and so on in the equation of a line.

The slope-intercept form of the equation of a line

The equation of a line may be written in the form

$$y = mx + c$$

This is called the slope-intercept form of the equation of a line.

When the equation of a line is written this way, we can immediately read the slope or gradient of the line and its y-intercept. The slope is m and the y-intercept is c.

The intercept form of the equation of a line

The equation of a line may be written in the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

This is called the intercept form of the equation of a line.

When the equation of a line is written this way, the x-intercept is a and the y-intercept is b.

5.12.3 How to find the equation of a line

A line is defined by two points or a point and its gradient.

Finding the equation of a line given its gradient and a point on the line

Given the gradient m of a line and a point $P(x_1, y_1)$ on the line, we can find the equation of the line as follows.

If a general point on the line has coordinates (x, y) then we have

$$\frac{y - y_1}{x - x_1} = m \quad \text{or} \quad y - y_1 = m(x - x_1)$$

From this, we can get the equation of the line.

For example, if a line has gradient 2 and passes through the point $(3, 4)$, then we can find its equation by writing

$$\begin{aligned} \frac{y-4}{x-3} &= 2 \\ y - 4 &= 2(x-3) \\ y - 4 &= 2x - 6 \\ y &= 2x - 2 \end{aligned}$$

Finding the equation of a line given two points on the line

Given two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ on a line, we can find the equation of the line by first using the two points to find the gradient. After that, we use one of the points and the gradient we found to find the equation of the line.

The gradient of the line is

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Using the point $P(x_1, y_1)$ and the gradient, we can find the equation of the line by writing

$$\begin{aligned} \frac{y - y_1}{x - x_1} &= \frac{y_2 - y_1}{x_2 - x_1} \\ y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \end{aligned}$$

For example, to find the equation of the line that passes through $(1, 2)$ and $3, 5$, we first find the gradient as

$$\frac{5-2}{3-1} = \frac{3}{2}$$

Using this gradient and the point $(1, 2)$, we can find the equation of the line as

$$\begin{aligned}\frac{y-2}{x-1} &= \frac{3}{2}\\ y-2 &= \frac{3}{2}(x-1)\\ y-2 &= \frac{3}{2}x-\frac{3}{2}\\ y &= \frac{3}{2}x-\frac{3}{2}+2\\ y &= \frac{3}{2}x+\frac{1}{2}\end{aligned}$$

5.12.4 The distance between two points

Given two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, the distance between them is given by the formula

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

This distance is the same as the length of the line segment between the two points.

For example, to find the distance of the between $C(2,4)$ and $D(6,1)$, we use the formula to get

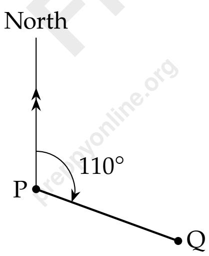
$$\begin{aligned}\sqrt{(2-6)^2+(4-1)^2} &= \sqrt{(-4)^2+3^2}\\ &= \sqrt{16+9}\\ &= \sqrt{25}\\ &= 5 \text{ units}\end{aligned}$$

5.13 Bearings

Bearings are measured clockwise from due north.

For example, if the bearing or Q from P is 110°, that means that if we stand at P and start measuring clockwise from the North, Q is going to be on the line that is 110° from the North.

The figure below shows the locations of P and Q.



By convention, bearings are written with three digits. Hence, a bearing of 80° is written as 080°.

5.13.1 Back bearings

Given the bearing of Q from P, we can find the bearing of P from Q, also known as the back bearing, as follows:

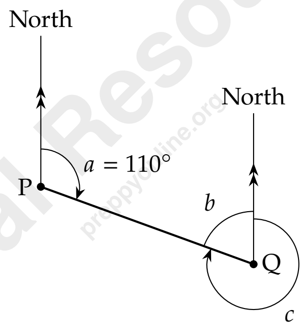
If the given bearing, x, is such that $x < 180°$, add 180° to it.

If the given bearing, x, is such that $x \geqslant 180°$, subtract 180° from it.

For example, if the bearing of Q from P is 110°, then, using the formula described above, the bearing of P from Q is $110° + 180° = 290°$.

But we could also find the back bearing by drawing a diagram and using the relationships between the angles to figure it out.

The information given can be represented with the diagram below.



The diagram shows the bearing of Q from P as angle a, while the bearing of P from Q is represented by angle c.

Because angles a and b are co-interior angles, they add up to 180°. So,

$$\begin{aligned}a+b &= 180°\\ 110°+b &= 180°\\ b &= 180°-110°\\ b &= 70°\end{aligned}$$

Also, because angles b and c are angles at a point, we have

$$\begin{aligned}b+c &= 360°\\ 70°+c &= 360°\\ c &= 360°-70°\\ c &= 290°\end{aligned}$$

Hence, the bearing of P from Q is 290°.

5.14 Vectors

Vectors are usually written in component form. For example, $\mathbf{v} = \begin{pmatrix} a \\ b \end{pmatrix}$. The entries in the vector are called components.

In $\mathbf{v} = \begin{pmatrix} a \\ b \end{pmatrix}$, a is called the x component while b is called the y component.

5.14.1 Rules of vector algebra

Vectors can be added, subtracted, and multiplied by numbers.

Addition of vectors

To add two vectors, we add the corresponding components.

For example, if $\mathbf{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$$\begin{aligned} \mathbf{v} + \mathbf{w} &= \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} -2+1 \\ 3+2 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 5 \end{pmatrix} \end{aligned}$$

Subtraction of vectors

Subtraction of vectors is also done component by component.

For example, if $\mathbf{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$$\begin{aligned} \mathbf{v} - \mathbf{w} &= \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} -2-1 \\ 3-2 \end{pmatrix} \\ &= \begin{pmatrix} -3 \\ 1 \end{pmatrix} \end{aligned}$$

Scalar multiplication

Vectors can be multiplied by numbers. The number by which a vector is multiplied is called a scalar.

If the vector $\mathbf{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ is multiplied by the scalar k, we get

$$k\mathbf{v} = \begin{pmatrix} ka \\ kb \end{pmatrix}$$

That is, each component of the vector is multiplied by the number.

For example,

$$2\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2(-3) \\ 2(4) \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$$

5.14.2 The length of a vector

Given a vector $\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$, the length of $\mathbf{v}$, denoted $|\mathbf{v}|$, is given by the formula

$$|\mathbf{v}| = \sqrt{x^2 + y^2}$$

For example, the length of the vector $\mathbf{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ is

$$|\mathbf{a}| = \left|\begin{pmatrix} -2 \\ 3 \end{pmatrix}\right| = \sqrt{(-2)^2 + 3^2} = \sqrt{4+9} = \sqrt{13} \text{ units}$$

5.15 Trigonometry

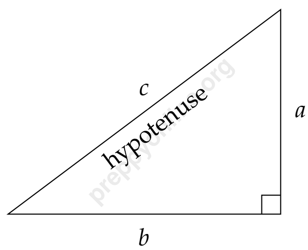
Trigonometry is about the measurement of the sides and angles of triangles.

Right-angled triangles are of special interest here because, the the basic trigonometric ratios (sin, cos, tan) are easy to define in terms of the sides of such a triangle.

5.15.1 Pythagoras' theorem

Given a right angled triangle like the one below, the longest side is called they hypotenuse.

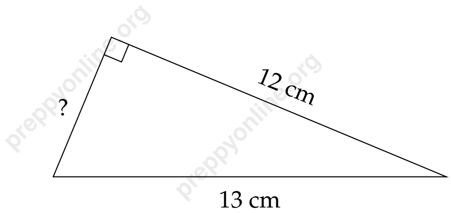
The hypotenuse is always opposite the 90° angle in the triangle.



Pythagoras' theorem states that, if the length of the hypotenuse is c and the lengths of the other sides are a and b, then the relationship between the lengths is

$$c^2 = a^2 + b^2$$

For example, since the triangle below is a right-angled triangle with hypotenuse of length 13 cm, we can use Pythagoras' theorem to find the third side as follows.



Let the length of the third side be x. Then, by Pythagoras' theorem,

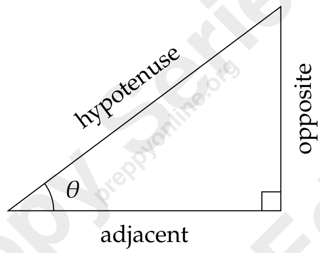
$$
\begin{aligned}
x^2 + 5^2 &= 13^2 \\
x^2 + 25 &= 169 \\
x^2 &= 169 - 25 \\
x^2 &= 144 \\
x &= 12
\end{aligned}
$$

Thus, the length of the third side is 12 cm.

5.15.2 SOH-CAH-TOA

When one of the angles in a right-angled triangle is of interest, we use that as a reference angle to name the sides of the triangle.

The side opposite the reference angle is called the opposite side, while the side next to the reference angle (that is not the hypotenuse) is called the adjacent side. The longest side, the hypotenuse, remains the hypotenuse.



With these labels, the basic trigonometric ratios as follows.

The sine of θ is given by

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

The cosine of θ is given by

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

The tangent of θ is given by

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

These are shortened by the mnemonic SOH-CAH-TOA.

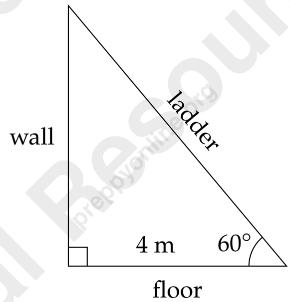
SOH means $S = \frac{O}{H}$, where S stands for sin, O stands for "opposite," and H stands for "hypotenuse."

CAH means $C = \frac{A}{H}$, where C stands for cos, A stands for "adjacent," and H stands for "hypotenuse."

TOA means $T = \frac{O}{A}$, where T stands for tan, O stands for "opposite," and A stands for "adjacent."

A ladder leans against a vertical wall so that it makes an angle of 60° with the ground. If the foot of the ladder is 4 metres from the wall, find the length of the ladder.

The sketch below shows the relationship between the ladder, the wall, and the floor.



Let the length of the ladder be x. Then, from the sketch,

$$\cos 60^\circ = \frac{4\text{ m}}{x}$$

Thus,

$$x = \frac{4\text{ m}}{\cos 60^\circ}$$

If we know the value of $\cos 60^\circ$, we can use it to calculate the length of the ladder.

Using a calculator, we find $\cos 60^\circ$ to be $\frac{1}{2}$. Hence, the length of the ladder is

$$\frac{4\text{ m}}{\frac{1}{2}} = 2(4\text{ m}) = 8\text{ m}$$

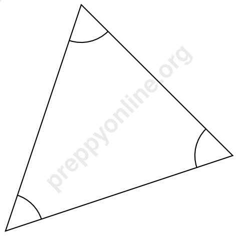
5.16 Plane geometry

5.16.1 Types of angles

Acute angles

An acute angle is an angle that measures greater than 0° and less than 90°.

Acute angles are known for their sharpness or pointedness.

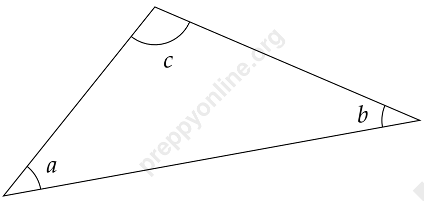


All the angles in the triangle above are examples of acute angles.

Obtuse angles

An obtuse angle is an angle that measures greater than 90° and less than 180°.

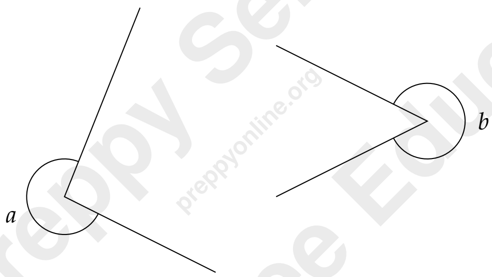
Angle c below is an obtuse angle, while angles a and b are acute angles.



Reflex angles

A reflex angle is an angle that measures greater than 180° and less than 360°.

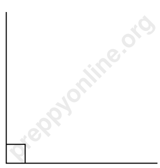
Examples of reflex angles are angles a and b below.



Right angles

A right angle is an angle that measures exactly 90°.

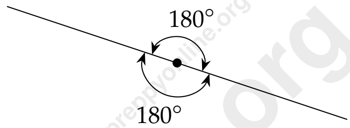
It is the angle at the corner of a square or a perfect "L" shape.



Straight angles

A straight angle is an angle that measures 180°.

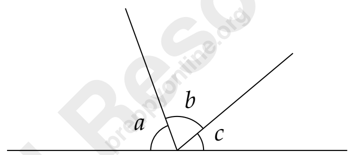
They are angles about a point on either side of a straight line.



Angles on a straight line

Since a straight angle measures exactly 180°, when it is divided up, the measures of the parts sum up to 180°.

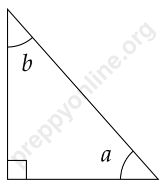
For example, angles a, b, and c add up to 180° as they are angles on a straight line.



Complementary angles

Two angles are called complementary angles if they add up to 90°.

An example of complementary angles is the two acute angles in a right-angled triangle.



Since the angles in a triangle add up to 180°,

$$a + b + 90° = 180°$$

Hence,

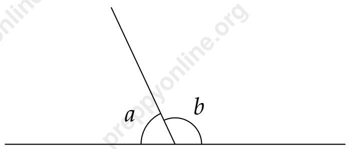
$$a + b = 180° - 90° = 90°$$

Therefore, a and b are complementary angles.

Supplementary angles

Supplementary angles are a pair of angles whose measures add up to 180°.

An example of supplementary angles are two angles on a straight line like angles a and b below.

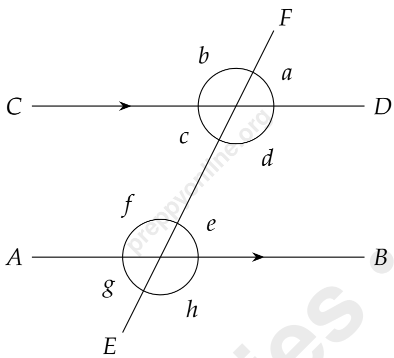


5.16.2 Angles in parallel lines

In the diagram below, lines AB and CD are parallel, as indicated by the arrows on them.

Line EF cuts through lines AB and CD. It is called a transversal.

A transversal is a line that crosses two or more lines in a plane.



Whenever a transversal crosses a pair of parallel lines, it creates 8 angles with special relationships between them.

We shall discuss the relationships between the 8 angles a, b, c, d, e, f, g, h below.

Alternate angles

Angles on opposite sides of the transversal and between the parallel lines are equal.

These are called alternate angles.

In the diagram above, c and e are alternate angles. So are d and f.

Corresponding angles

Angles in the same relative position at each intersection are equal.

For example, the angle at the top-left of the intersection at the top, b, is equal to the angle at the top-left of the intersection at the bottom, f.

Co-interior angles

Angles on the same side of the transversal and between the parallel lines add up to 180°.

These are called co-interior or allied angles. They are also called consecutive interior angles.

Examples are angles d and e in the diagram.

c and f are also examples of co-interior angles.

Vertically opposite angles

Angles opposite each other at the intersections are equal.

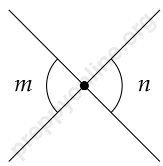
These are called vertically opposite angles.

Examples include a and c in the diagram above.

Angles b and d are also vertically opposite and therefore equal.

Vertically opposite angles are called "vertically" opposite because they are opposite each other across their common vertex—the point where two lines meet or cross—and not because they are oriented up and down. Thus, here, "vertical" is the adjectival form of "vertex."

In the diagram below, angles m and n are vertically opposite angles even though they are not oriented up and down.



5.17 Statistics

5.17.1 Types of data

Quantitative data

Quantitative data are values that can be measured numerically. Examples include

- length (as it can be measured in numbers like metres, centimetres, inches, etc.)
- weight (as it can be measured in numbers like kilograms, pounds, etc.)
- time (as it can be measured in numbers like hours, minutes, seconds, etc.)
- speed (as it can be measured in numbers like metres per second, kilometres per hour, etc.)
- temperature (as it can be measured in numbers like degrees Celsius, degrees Fahrenheit, etc.)

Qualitative data

Qualitative data are values that describe qualities or attributes rather than numerical measurements. Such data can typically be arranged into categories or groups. Examples include

- colour (red, blue, green, etc.)
- marital status (single or married)
- nationality (Ghanaian, Nigerian, Japanese, etc.)

5.17.2 Measures of central tendency

These give a single number by which the data can be represented. Examples include the mean, the median, and the mode.

The mean

The mean of a list of numbers is given by the sum of the numbers divided by how many numbers there are.

For example, the mean of the numbers 1, 1, 2, 2, 2, 2, 3, 3, 4 is

$$\frac{1+1+2+2+2+2+3+3+4}{9} = \frac{20}{9}$$

The median

The median is the middle number when data is arranged in ascending or descending order.

For example, in the data 1, 1, 2, 2, 2, 2, 3, 3, 4, the median is 2 because 2 is the middle number when the numbers are arranged in ascending or descending order.

The mode

Given some data, such as a list of numbers, the mode is the data that occurs most frequently.

For example, in the data 1, 1, 2, 2, 2, 2, 3, 3, 4,

1 appears 2 times.

2 appears 4 times.

3 appears 2 times.

4 appears once.

Hence, the mode is 2 as 2 is the data that appears most frequently.

5.17.3 Measures of dispersion

These give an idea of spread out the data is.

Examples include the range and the standard deviation.

Range

Given a list of numbers, the range is the difference between the largest and the smallest.

For example, in the list 1, 1, 2, 2, 2, 2, 3, 3, 4, the range is

$$4 - 1 = 3$$

since the largest number in the list is 4 and the smallest number is 1.

5.18 Loci

A locus is a path traced by a moving point.

There are many rules by which a point could move, and each of them gives a locus.

We shall mention some important examples below.

5.18.1 Examples of loci

The circle

If a point moves so that it is always the same distance from a given point, the moving point will trace a circle with the given point as its centre.

The perpendicular bisector

The set of points that are equidistant from two given points will trace the perpendicular bisector of the line joining the two given points.

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