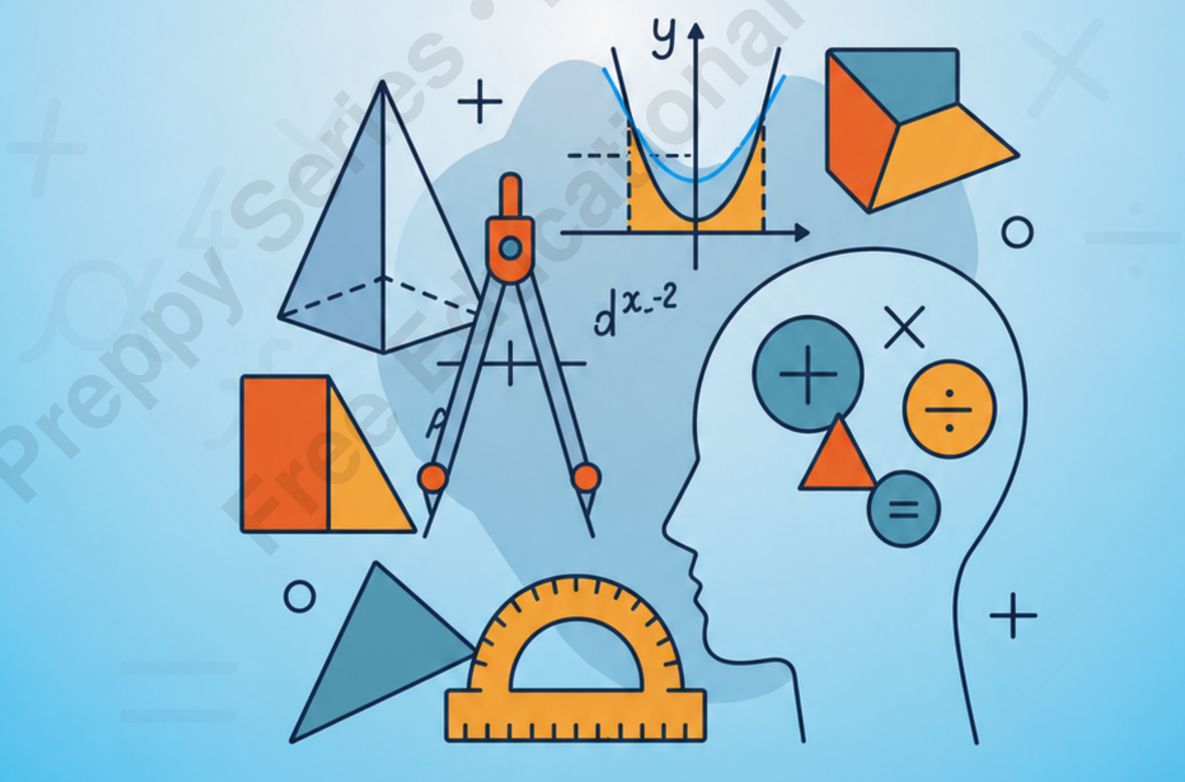
Preppy Series

MATHEMATICS

for Junior High Schools

BECE Past Questions & Solutions

Arranged by Year



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Mathematics

for Junior High Schools

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Compiled by

preppyonline.org

2026 Edition

Preface

This book is designed to be a helpful resource for junior high school students, especially those preparing for the Basic Education Certificate Examination (BECE) of the West African Examinations Council (WAEC).

It features

- a collection of complete BECE Mathematics past papers—both objective and essay-type questions—arranged by year
- detailed and thoughtfully explained solutions
- short notes on important facts and methods frequently used on the exams

Our philosophy is simple: learning Mathematics should go beyond memorizing procedures or blindly applying algorithms and tricks. True understanding comes from engaging with problems, exploring different approaches, and appreciating how and why solutions work. For this reason, each solution is presented

- in a step by step fashion
- with clear reasoning
- with multiple perspectives where appropriate
- and with an emphasis on building strong conceptual foundations

Special attention has been given to clarity, with well-drawn diagrams and structured workings that carry the student along through the solution process.

In line with the goal of widening access to quality educational materials, this resource is available at preppyonline.org as a free PDF download. It is our hope that students, teachers, and independent learners alike will find it useful not just for exam preparation, but for developing confidence and deeper insight into Mathematics.

Comments, suggestions, and other enquiries may be sent to preppyonline@gmail.com

Contents

Chapter 1

2026 Paper 1

1. Arrange the following: $\frac{3}{4}, 0.8, \frac{5}{8}, 0.65$ in descending order.

A. $0.8, \frac{3}{4}, 0.65, \frac{5}{8}$

B. $\frac{5}{8}, 0.8, \frac{3}{4}, 0.65$

C. $0.65, \frac{5}{8}, \frac{3}{4}, 0.8$

D. $\frac{3}{4}, \frac{5}{8}, 0.8, 0.65$

2. Mr. Mensah left his house at 9:45 am and reached his village at 4:15 pm. Find the time spent.

A. 5 hours 30 minutes

B. 8 hours

C. 6 hours 30 minutes

D. 7 hours

3. In a test, $\frac{2}{3}$ of the learners passed. If the number of learners who failed was 69, how many passed?

A. 23

B. 138

C. 46

D. 207

4. Factorize: $3a^2b - 9ab^2$.

A. $3ab(a - 3b)$

B. $ab(3a - b)$

C. $3ab(b - 3a)$

D. $ab(3b - a)$

5. Find the volume of a cube with side 5 m.

A. 10 m^3

B. 75 m^3

C. 25 m^3

D. 125 m^3

6. A trader sold an article for GH¢ 126.00 marking a profit of 20%. Find the cost price of the item.

A. GH¢ 100.50

B. GH¢ 105.00

C. GH¢ 100.80

D. GH¢ 151.20

7. Write 0.000437 in standard form.

A. 4.37×10^{-4}

B. 4.37×10^{-3}

C. 4.37×10^{4}

D. 4.37×10^{3}

8. Mary receives a commission of 15% on articles sold in a week. If her commission was GH¢ 60.00, how much sales did she make?

A. GH¢ 900.00

B. GH¢ 90.00

C. GH¢ 400.00

D. GH¢ 40.00

9. Simplify: $\sqrt{75} - \sqrt{18} - \sqrt{3} + \sqrt{2}$.

A. $4\sqrt{3} - 2\sqrt{2}$

B. $3\sqrt{3} - 2\sqrt{2}$

C. $4\sqrt{3} + 2\sqrt{2}$

D. $3\sqrt{3} + 2\sqrt{2}$

10. Theresa was asked to select at random a letter from the word **HAPPY**. What is the probability that she selects the letter **P**?

A. $\frac{1}{5}$

B. $\frac{1}{2}$

C. $\frac{2}{5}$

D. $\frac{3}{5}$

11. Find the image of the point $(2, -3)$ under the transformation $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y-2 \end{pmatrix}$.

A. $(2, 1)$

B. $(2, -1)$
C. $(2, -5)$
D. $(2, 5)$

12. A square of area 144 cm^2 has the same perimeter as an equilateral triangle. Find the length of a side of the triangle.
A. 10 cm
B. 16 cm
C. 14 cm
D. 18 cm

13. Solve: $7 - 2x > 15 - 4x$.
A. $x < -4$
B. $x < 4$
C. $x > -4$
D. $x > 4$

14. Antwi has 20 mangoes and 20% are rotten. How many of them are **not** rotten?
A. 4
B. 12
C. 8
D. 16

15. A class of 42 learners shared some oranges and **each** received 11. If 22 learners shared the same number of oranges **equally**, how many will each get?
A. 15
B. 21
C. 20
D. 22

16. Make y the subject of the relation $p = \frac{r - 4y}{3}$.
A. $y = \frac{1}{4}(3p - r)$
B. $y = \frac{1}{4}(r - 3p)$
C. $y = \frac{1}{4}(r + 3p)$
D. $y = \frac{1}{4}(r + p)$

17. A number is chosen at random from the set $P = \{1, 2, 3, 4, 5, \ldots, 10\}$. What is the probability that the number is greater than 3?
A. $\frac{4}{5}$
B. $\frac{3}{10}$
C. $\frac{7}{10}$
D. $\frac{1}{5}$

18. Kofi paid an interest of GH¢ 30.00 on a loan he took for 4 years. If the rate was 3% per annum simple interest, find the amount borrowed.
A. GH¢ 360.00
B. GH¢ 250.00
C. GH¢ 280.00
D. GH¢ 90.00

19. A cyclist travelling at 20 km/h covered a distance in 35 minutes. What time will it take to cover the same distance travelling at 28 km/h?
A. 16 minutes
B. 48 minutes
C. 25 minutes
D. 49 minutes

20. Which of the following is **not** a composite number?
A. 24
B. 41
C. 39
D. 65

21. Kofi had 150 birds. He sold 26 of them and kept the rest equally in 4 cages. How many birds were kept in **each** cage?
A. 30
B. 35
C. 31
D. 36

22. If $(x + 2) : (x - 2) = 1 : 2$, find the value of x.
A. -2
B. -6
C. -3
D. -35

23. Given that $\mu = \{1, 2, 3, \ldots, 10\}$ and $M = \{2, 3, 5, 7\}$, where M is a subset of μ, list the members in μ that are **not** in M.
A. $\{1, 4, 6\}$
B. $\{2, 4, 6, 8, 10\}$
C. $\{2, 4, 6, 8\}$
D. $\{1, 4, 6, 8, 9, 10\}$

24. If $2^{2m} = 8$, find the value of m.
A. 2.0
B. 1.0
C. 1.5
D. 0.5

25. A boy spends $\frac{1}{4}$ of his pocket money on books and $\frac{1}{3}$ on pens. What fraction remains?

A. $\frac{5}{6}$
B. $\frac{5}{12}$
C. $\frac{7}{12}$
D. $\frac{1}{6}$

26. Two sets which have the same number of members are . . . sets.

A. equal
B. intersecting
C. equivalent
D. union

27. Change 25% to a fraction in its lowest form.

A. $\frac{1}{2}$
B. $\frac{1}{8}$
C. $\frac{1}{4}$
D. $\frac{5}{6}$

28.

$$\begin{array}{cccccc} x & 1 & 2 & 3 & 4 & 5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & 3 & 6 & 9 & 12 & 15 \end{array}$$

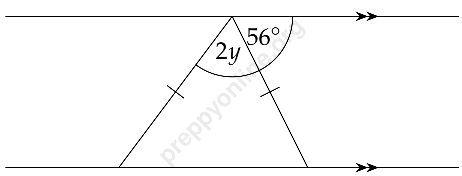
What is the rule for the mapping?

A. $x \longrightarrow 3x$
B. $x \longrightarrow 4 - x$
C. $x \longrightarrow x + 2$
D. $x \longrightarrow 2x + 1$

29. Find the circumference of a circle whose area is 100π cm^2.

A. 5π cm
B. 15π cm
C. 10π cm
D. 20π cm

30.



NOT DRAWN TO SCALE

Find the value of y in the diagram.

A. 60°
B. 30°
C. 34°
D. 28°

31. Given the sequence $-9, -5, m, 3, 7, 11$, find the value of m.

A. -3
B. -1
C. -2
D. 1

32. The marks obtained by 11 learners in a test are: 2, 5, 5, 6, 7, 7, 8, 8, 8, 9, 10. What is the modal mark?

A. 2
B. 8
C. 7
D. 9

33. If $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, evaluate $6\mathbf{b} + 2\mathbf{a}$.

A. $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$
B. $\begin{pmatrix} -6 \\ 8 \end{pmatrix}$
C. $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$
D. $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$

34. Evaluate: $p^2(q - 1)$ when $p = 2$ and $q = \frac{3}{4}$.

A. -2
B. 1
C. -1
D. 2

35. Ivy and Abbey share an amount of GH¢ 30.00 in the ratio 3 : 2 respectively. Find Abbey's share.

A. GH¢ 10.00
B. GH¢ 15.00
C. GH¢ 12.00
D. GH¢ 18.00

Mark	0	1	2	3	4	5
Frequency	1	2	7	5	4	3

The table shows marks scored by a group of learners in a test. Use this information to answer questions 36 and 37.

36. Find the median mark.

A. 1
B. 3
C. 2
D. 4

37. Find the probability that a learner selected at random scored 2.

A. $\frac{1}{22}$
B. $\frac{2}{22}$
C. $\frac{7}{22}$
D. $\frac{4}{22}$

38. If $\frac{3}{4}x = 2 + \frac{1}{4}$, find the value of x.

A. 1
B. 4
C. 3
D. 5

39. The product of three numbers is 90. If two of the numbers are 6 and 3, find the **third** number.

A. 18
B. 9
C. 15
D. 5

40. Simplify: $x - 5(3 - 2x) - 12x + 7$.

A. $x + 8$
B. $-21x + 8$
C. $-21x - 8$
D. $-x - 8$

Chapter 2

Solutions to 2026 Paper 1

Answer key

1. A	9. A	17. C	25. B	33. B
2. C	10. C	18. B	26. C	34. C
3. B	11. C	19. C	27. C	35. C
4. A	12. B	20. B	28. A	36. B
5. D	13. D	21. C	29. D	37. C
6. B	14. D	22. B	30. C	38. C
7. A	15. B	23. D	31. B	39. D
8. C	16. B	24. C	32. B	40. D

Solutions

1. Answer: A

We are asked to arrange some numbers in descending order, that is, from largest to smallest. But it's not so straightforward to do that because some of the numbers are written as fractions while others are written as decimals:

$$\frac{3}{4}, 0.8, \frac{5}{8}, 0.65$$

If we wrote all the numbers as decimals, they would be easier to compare. Similarly, writing them all as percentages would make them easier to compare.

Writing $\frac{3}{4}$ as a percentage we get

$$\frac{3}{4} \times 100\% = \frac{3}{\cancel{4}_1} \times \cancel{100\%}^{25} = 75\%$$

Writing 0.8 as a percentage we get

$$0.8 \times 100\% = 80\%$$

Writing $\frac{5}{8}$ as a percentage we get

$$\frac{5}{8} \times 100\% = \frac{5}{\cancel{8}_2} \times \cancel{100\%}^{25} = 62.5\%$$

Writing 0.65 as a percentage we get

$$0.65 \times 100\% = 65\%$$

We thus have the following:

$$\frac{3}{4} = 75\% \quad 0.8 = 80\% \quad \frac{5}{8} = 62.5\% \quad 0.65 = 65\%$$

Using the percentages to arrange these numbers from largest to smallest, we get

$$0.8, \frac{3}{4}, 0.65, \frac{5}{8}$$

2. Answer: C

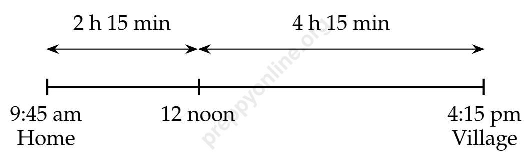
Mr. Mensah started his journey at 9:45 am and ended it at 4:15 pm. To find the the time spent on this journey, we need to calculate how long it is from 9:45 am to 4:15 pm.

We will look at two ways of solving this problem.

Method 1

If we divide the time Mr. Mensah spent on the journey into two parts: the part before 12 noon and the part after 12 noon, we can calculate how much time was spent on each of those parts and add them up to get the total.

From the sketch below, we see that the part before 12 noon was 2 hours 15 minutes long while the part after 12 noon was 4 hours 15 minutes long.



Adding up these time periods, we get

$$\begin{aligned} 2\text{ h } 15\text{ min} + 4\text{ h } 15\text{ min} &= (2+4)\text{ h} + (15+15)\text{ min} \\ &= 6\text{ h} + 30\text{ min} \\ &= 6\text{ h } 30\text{ min} \end{aligned}$$

Method 2

Mr. Mensah started his journey at 9:45 am.

Suppose he had ended his journey at 10:45 am. How long would he have spent on the journey?

1 hour.

What if he had ended his journey at 11:45 am? How long would he have spent on the journey?

2 hours.

What about 12:45 pm? 3 hours.

1:45 pm $\longrightarrow$ 4 hours

2:45 pm $\longrightarrow$ 5 hours

3:45 pm $\longrightarrow$ 6 hours

4:45 pm $\longrightarrow$ 7 hours

The time from 9:45 am to 4:45 pm is 7 hours but Mr. Mensah ended his journey at 4:15 pm, not 4:45 pm. That is, he ended his journey 30 minutes before 4:45 pm.

Thus, the total time spent on the journey is

$$7 \text{ hours} - 30 \text{ minutes} = 6 \text{ hours } 30 \text{ minutes}$$

3. Answer: B

If $\frac{2}{3}$ of the learners who took a test passed, then the fraction of learners who failed was

$$1 - \frac{2}{3} = \frac{3}{3} - \frac{2}{3} = \frac{1}{3}$$

If $\frac{1}{3}$ of the learners corresponds to 69, what does $\frac{2}{3}$ of the learners correspond to?

Since $\frac{2}{3} = 2 \times \frac{1}{3}$, $\frac{2}{3}$ of the learners corresponds to

$$2 \times 69 = 138$$

Thus, the number of learners who passed is 138.

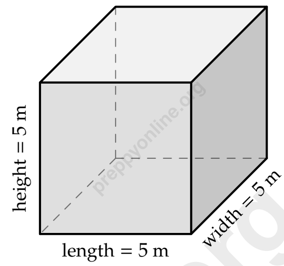
4. Answer: A

To factorize $3a^2b - 9ab^2$, we bring out the common factor of $3ab$ to get

$$3a^2b - 9ab^2 = 3ab(a - 3b)$$

5. Answer: D

Cubes are unique because their three dimensions—length, width, and height—are equal.



Therefore, if we know one of the sides, say the length, we can use it to get the volume with the formula (Section 5.7.3)

$$\text{volume} = \text{length} \times \text{length} \times \text{length} = (\text{length})^3$$

Since the given cube has side length 5 m, its volume is

$$5 \text{ m} \times 5 \text{ m} \times 5 \text{ m} = 125 \text{ m}^3$$

6. Answer: B

If by selling the item at GH¢ 126.00 the trader made a profit of 20%, then GH¢ 126.00 is 120% of the cost price of the item.

If 120% of the cost price is GH¢ 126.00, what is 100% of it?

Letting the cost price be GH¢ x, we have

$$120\% : \text{GH¢ } 126 = 100\% : \text{GH¢ } x$$

This means

$$\frac{120}{126} = \frac{100}{x}$$

Cross-multiplying and solving for x we get

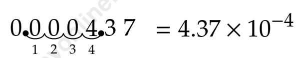
$$120x = 100 \times 126$$
$$x = \frac{100 \times 126}{120}$$
$$x = 105$$

Thus, the cost price of the item is GH¢ 105.00.

7. Answer: A

To write 0.000437 in standard form (Section 5.2.3), we need to write it in the form $a \times 10^n$ where $1 \leqslant a < 10$ and n is an integer.

This means moving the decimal point to the right till it is after the first non-zero number, while counting how many times we had to move it. Doing this and counting how many times the point had to be moved, we get



Because 0.000437 is less than 1, we had to move the decimal point to the right and the exponent of 10 in the standard form expression is negative. Had the number been greater than one, we would have had to move the decimal point to the left or leave it where it was and the exponent of 10 would have been zero or positive. You may read Section 5.2.3 for more details.

8. Answer: C

Whatever sales Mary makes, she receives 15% of it as commission. Hence, if she received GH¢ 60.00 as commission, that GH¢ 60.00 must have been 15% of the sales she made. If so, how much sales did she make?

The question we have to answer is: If 15% of a number is 60, what is that number?

Asking for the number is the same as asking for 100% of the number.

Letting t represent the sales she made, we may write the following equivalent ratios:

$$15\% : 60 = 100\% : t$$

This means

$$\frac{15}{60} = \frac{100}{t}$$

Cross-multiplying and solving for t we get

$$\begin{aligned} 15t &= 100 \times 60 \\ t &= \frac{100 \times 60}{15} \\ t &= 400 \end{aligned}$$

Hence, the sales Mary made was GH¢ 400.00.

9. Answer: A

To simplify $\sqrt{75} - \sqrt{18} - \sqrt{3} + \sqrt{2}$, we apply the rules of surds (Section 5.10).

Whenever possible, we will try to write the number under the square root as a product of a perfect square and another number.

For example, 75 can be written as the product of the perfect square 25 and the number 3:

$$75 = 25 \times 3$$

while 18 can be written as the product of the perfect square 9 and the number 2:

$$18 = 9 \times 2$$

Using these two facts, we may simplify the given expression as

$$\begin{aligned} &\sqrt{75} - \sqrt{18} - \sqrt{3} + \sqrt{2} \\ &= \sqrt{25 \times 3} - \sqrt{9 \times 2} - \sqrt{3} + \sqrt{2} \\ &= \sqrt{25}\sqrt{3} - \sqrt{9}\sqrt{2} - \sqrt{3} + \sqrt{2} \\ &= 5\sqrt{3} - 3\sqrt{2} - \sqrt{3} + \sqrt{2} \\ &= 5\sqrt{3} - \sqrt{3} - 3\sqrt{2} + \sqrt{2} \\ &= 4\sqrt{3} - 2\sqrt{2} \end{aligned}$$

10. Answer: C

Since Theresa selects a letter at random, each of the letters has an equal chance of being selected.

The probability of selecting any letter is

$$\frac{\text{how many times the letter appears in the word}}{\text{the number of letters in the word}}$$

Since there are 5 letters in the word HAPPY and two of them are the letter P, the probability of selecting the letter P is

$$\frac{2}{5}$$

11. Answer: C

Find the image of the point $(2, -3)$ under the transformation $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ y-2 \end{pmatrix}$.

To find the image of the point $(2, -3)$ under the transformation $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ y-2 \end{pmatrix}$, we need to take note of what its x component is and what its y component is.

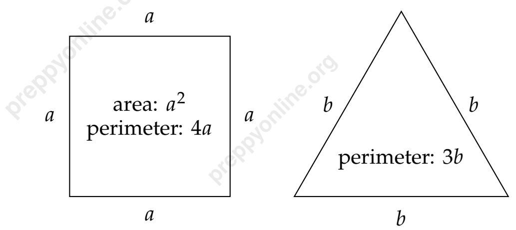
The x component of $(2, -3)$ is 2, and its y component is -3. Hence, under the given transformation,

$$\begin{pmatrix} 2 \\ -3 \end{pmatrix} \to \begin{pmatrix} 2 \\ -3-2 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

Thus, under the transformation, the image of the point $(2, -3)$ is $(2, -5)$.

12. Answer: B

We draw a sketch of both a square and an equilateral triangle (a triangle with all three sides equal) below.



The area of a square of side a is a^2. Hence, if the area of the square is 144 cm^2, we have

$$\begin{aligned} a^2 &= 144\ \text{cm}^2 \\ \sqrt{a^2} &= \sqrt{144\ \text{cm}^2} \\ a &= 12\ \text{cm} \end{aligned}$$

Thus, the length of a side of the square is 12 cm.

If the length of the side of a square is 12 cm, what is its perimeter? The perimeter of a square is 4 times the length of its side. Hence, the perimeter of the square is

$$4 \times 12\ \text{cm} = 48\ \text{cm}$$

If the perimeter of the square is equal to the perimeter of an equilateral triangle, then the perimeter of the equilateral triangle is 48 cm.

But the perimeter of an equilateral triangle is 3 times the length of its side. Hence, if its perimeter is 48 cm, the length of its side must be

$$\frac{48\ \text{cm}}{3} = 16\ \text{cm}$$

13. Answer: D

To solve the inequality $7 - 2x > 15 - 4x$, we group like terms and simplify to get

$$\begin{aligned} 7 - 2x &> 15 - 4x \\ -2x + 4x &> 15 - 7 \\ 2x &> 8 \\ \frac{2x}{2} &> \frac{8}{2} \\ x &> 4 \end{aligned}$$

14. Answer: D

To find the number of oranges that are not rotten, we may first find the number of oranges that are rotten and subtract that from the total number of oranges.

If Antwi has 20 mangoes and 20% are rotten, how many are rotten?

$$\begin{aligned} \text{number of rotten oranges} &= 20\% \text{ of } 20 \text{ oranges} \\ &= \frac{20}{100} \times 20 \text{ oranges} \\ &= 4 \text{ oranges} \end{aligned}$$

If 4 of the oranges are rotten, then the number that are not rotten is

$$20 - 4 = 16$$

15. Answer: B

We can first find the total number of oranges shared by the 42 learners and divide that number by 22 to see how many each will get.

If 42 learners shared a number of oranges and each received 11, then the total number of oranges they shared was

$$42 \times 11 = 462$$

If 22 learners share 462 oranges equally, each will get

$$\frac{462}{22} = 21$$

16. Answer: B

To make y the subject of the given relation, we apply the rules of algebra to rearrange the equation to isolate y.

$$\begin{aligned} p &= \frac{r - 4y}{3} \\ 3p &= r - 4y \\ 4y &= r - 3p \\ y &= \frac{r - 3p}{4} \end{aligned}$$

Since the answer options are written with the fraction factored out, we also write our answer that way to get

$$\frac{r - 3p}{4} = \frac{1}{4}(r - 3p)$$

17. Answer: C

The sample space or the set of all possible outcomes is $P = \{1, 2, 3, 4, 5, \ldots, 10\}$. This set has 10 elements.

Out of these 10 possible outcomes, the set of outcomes that are greater than 3 is $\{4, 5, 6, 7, 8, 9, 10\}$. This set has 7 elements.

Hence, the probability that a number chosen at random from the set P is greater than 3 is

$$\frac{7}{10}$$

18. Answer: B

We are given the following:

a simple interest of GH¢ 30.00,

a rate of 3% per annum,

and a time of 4 years.

We are not given the principal or the amount borrowed. However, using the formula for simple interest, we can deduce this amount.

The formula for simple interest (Section 5.4) is

$$\text{simple interest} = \text{principal} \times \text{rate} \times \text{time}$$

Substituting the given values into the formula gives

$$\text{GH¢ } 30 = \text{principal} \times 3\% \times 4$$
$$\text{GH¢ } 30 = \text{principal} \times \frac{3}{100} \times 4$$

Multiplying both sides by the reciprocal of $\frac{3}{100} \times 4$ to isolate "principal" gives

$$\begin{aligned} \text{principal} &= \text{GH¢ } 30 \times \frac{100}{3} \times \frac{1}{4} \\ &= \text{GH¢ } 250.00 \end{aligned}$$

19. Answer: C

Travelling at 20 km/h, the cyclist covered a certain distance in 35 minutes. If his speed is increased to 28 km/h, he will use less time to cover the same distance. How do we find this time?

We shall look at two ways of solving the problem.

Method 1

Since the distance the cyclist travels at both speeds is the same, we can find the distance travelled at the two speeds and equate them to each other.

The formula for distance is

$$\text{distance} = \text{speed} \times \text{time}$$

At 20 km/h, he covered the distance in 35 minutes. Hence, the distance travelled was

$$\text{distance} = 20 \text{ km/h} \times 35 \text{ minutes}$$

Let the time taken to cover the distance at 28 km/h be t minutes. Then, the distance travelled at 28 km/h is

$$\text{distance} = 28 \text{ km/h} \times t \text{ minutes}$$

Equating the two distances, we get

$$\begin{aligned} 28 \text{ km/h} \times t \text{ minutes} &= 20 \text{ km/h} \times 35 \text{ minutes} \\ 28t &= 20 \times 35 \\ t &= \frac{20 \times 35}{28} \\ t &= 25 \end{aligned}$$

Thus, when the cyclist's speed is increased to 28 km/h, he covers the distance in 25 minutes.

Method 2

As the speed of the cyclist increases, the amount of time he uses to travel the distance decreases. And as his speed decreases, the amount of time he uses to travel the distance increases. Thus, there is an inverse relationship between his speed and the amount of time he uses.

Using the information given in the question, and letting t be the time the cyclist takes on the journey when travelling at 28 km/h, we can write the following:

$$20 \text{ km/h} \to 35 \text{ min} \quad \text{while} \quad 28 \text{ km/h} \to t \text{ min}$$

Using the inverse relationship between the cyclist's speed and the time he takes, we may write the following:

$$\frac{20}{28} = \frac{t}{35}$$

Cross-multiplying and solving for t we get

$$\begin{aligned} 20 \times 35 &= 28 \times t \\ t &= \frac{20 \times 35}{28} \\ t &= 25 \end{aligned}$$

Thus, when travelling at 28 km/h, the cyclist uses 25 minutes to complete his journey.

An important point to note when using this method is that, because of the inverse relationship between speed and time, when translating the relationship below into an equation, we swap the order of the quantities:

$$20 \text{ km/h} \to 35 \text{ min} \quad \text{while} \quad 28 \text{ km/h} \to t \text{ min}$$

becomes

$$\frac{20}{28} = \frac{t}{35} \quad \text{not} \quad \frac{20}{28} = \frac{35}{t}$$

Forgetting to swap the order of the quantities will change the inverse relationship (proportion) between speed and time into a direct relationship (proportion), which is not correct.

20. Answer: B

We are asked to find which number in the list is not a composite number (Section 5.2.2).

A composite number is a positive number greater than 1 that has at least one divisor other than 1 and itself.

Another definition of a composite number is that it is a positive integer greater than 1 that is not prime.

Thus, we may answer the question by looking for the prime number in the list.

We will look at each of the numbers given and make a decision.

24 is not prime because it has factors other than 1 and itself. One of its factors is 2.

39 is not prime because it has factors other than 1 and itself. One of its factors is 3.

65 is not prime because ia has factors other than 1 and itself. One of its factors is 5.

41 is prime as its only factors are 1 and itself, 41. Hence, it is not a composite number.

21. Answer: C

After selling 26 of his 150 birds, the number of birds Kofi had left was

$$150 - 26 = 124$$

If he kept these remaining 124 birds in 4 cages so that each cage had the same number of birds, then the number of birds in each cage was

$$\frac{124}{4} = 31$$

22. Answer: B

Since equivalent ratios can also be written as equivalent fractions, we may rewrite $(x + 2) : (x - 2) = 1 : 2$ as

$$\frac{x+2}{x-2} = \frac{1}{2}$$

Cross-multiplying and solving for x gives

$$\begin{aligned} \frac{x+2}{x-2} &= \frac{1}{2} \\ 2(x+2) &= 1(x-2) \\ 2x + 4 &= x - 2 \\ 2x - x &= -2 - 4 \\ x &= -6 \end{aligned}$$

23. Answer: D

We may list the members of set μ and set M and compare them to see which members of μ are not in M.

$$\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \text{ and } M = \{2, 3, 5, 7\}$$

By inspection, the members of μ that are not in M are

$$\{1, 4, 6, 8, 9, 10\}$$

24. Answer: C

We are asked to solve for m in the equation $2^{2m} = 8$.

This is an exponential equation in which the expression on the left-hand side has base 2 and exponent $2m$. Hence, writing the right-hand side, 8, also with a base of 2, would simplify the equation.

Noticing that $8 = 2^3$, we have

$$\begin{aligned} 2^{2m} &= 8 \\ 2^{2m} &= 2^3 \end{aligned}$$

Since the bases on both sides are equal, we equate exponents to get

$$\begin{aligned} 2m &= 3 \\ m &= \frac{3}{2} \end{aligned}$$

Since the answer options are all written in decimals, we covert our answer into a decimal to get

$$\frac{3}{2} = 1.5$$

25. Answer: B

We can find the fraction remaining by subtracting the total fraction spent from one whole.

The fraction of money spent is

$$\frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12} = \frac{7}{12}$$

Therefore, the fraction remaining is

$$1 - \frac{7}{12} = \frac{12}{12} - \frac{7}{12} = \frac{5}{12}$$

26. Answer: C

Two sets which have the same number of members are called equivalent sets (Section 5.1.5). For example, $\{1, 2, 3\}$ and $\{4, 5, 6\}$ are equivalent sets because they both have 3 elements.

Equal sets are sets that contain the same elements (Section 5.1.5). For example, $\{1, 2, 3\}$ and $\{2, 3, 1\}$ are equal sets because they contain the same elements. This is because the order of the elements of a set does not matter.

Intersecting sets are sets that have at lease one element in common. For example, $\{1, 2, 3\}$ and $\{2, 3, 4\}$ are intersecting sets because they have the elements 2 and 3 in common.

The term "union sets" is not standard in mathematics. We instead refer to the union of two sets, which is the set of elements that are in either set or both sets. For example, the union of $\{1, 2, 3\}$ and $\{2, 3, 4\}$ is $\{1, 2, 3, 4\}$.

27. Answer: C

To change 25% into a fraction in its lowest terms, we first write the percentage as a fraction and then simplify the fraction.

$$25\% = \frac{25}{100} = \frac{\cancel{25}^{\,1}}{\cancel{100}_{\,4}} = \frac{1}{4}$$

28. Answer: A

$$\begin{array}{cccccc} x & 1 & 2 & 3 & 4 & 5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ y & 3 & 6 & 9 & 12 & 15 \end{array}$$

By inspection, we get y from x by multiplying x by 3. Hence, the rule of the mapping is $x \to 3x$.

29. Answer: D

The area of a circle with radius r is given by (Section 5.7.2)

$$\text{area} = \pi r^2$$

Substituting the given area into the formula, we get

$$100\pi \text{ cm}^2 = \pi r^2$$

Dividing both sides by π and solving for r gives

$$\begin{aligned} \frac{100\pi \text{ cm}^2}{\pi} &= \frac{\pi r^2}{\pi} \\ 100 \text{ cm}^2 &= r^2 \\ \sqrt{100 \text{ cm}^2} &= \sqrt{r^2} \\ 10 \text{ cm} &= r \\ r &= 10 \text{ cm} \end{aligned}$$

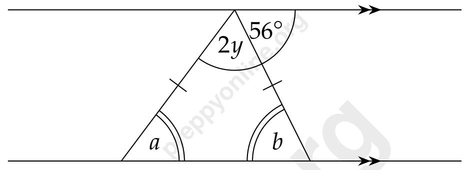
The circumference of a circle with radius r is given by the formula (Section 5.7.1)

$$\text{circumference} = 2\pi r$$

Substituting $r = 10$ cm into the formula gives

$$\text{circumference} = 2\pi \times 10 \text{ cm} = 20\pi \text{ cm}$$

30. Answer: C



Since the triangle is isosceles (as indicated by the markings on two of its sides), angles a and b are equal.

Also, angle b and the angle marked 56° are alternate angles. Hence, they are equal.

Since a and b are equal, both a and b measure 56°.

Because the angles in a triangle sum to 180°, we have

$$2y + a + b = 180^\circ$$

Substituting $a = 56^\circ$ and $b = 56^\circ$ into the equation above gives

$$\begin{aligned} 2y + 56^\circ + 56^\circ &= 180^\circ \\ 2y + 112^\circ &= 180^\circ \\ 2y &= 180^\circ - 112^\circ = 68^\circ \\ \frac{2y}{2} &= \frac{68^\circ}{2} \\ y &= 34^\circ \end{aligned}$$

31. Answer: B

By inspection, the terms of the sequence $-9, -5, m, 3, 7, 11$ increase by 4 at each step.

That is, given a term, the next term is obtained by adding 4 to it:

$$-9 \xrightarrow{+4} -5 \xrightarrow{+4} m \xrightarrow{+4} 3 \xrightarrow{+4} 7 \xrightarrow{+4} 11$$

Hence, we can get the value of m by adding 4 to -5, the term before m:

$$m = -5 + 4 = -1$$

32. Answer: B

The modal (Section 5.17.2) mark is the mark that appears most frequently.

By inspection, 8 is the mark that appears most frequently. It appears three times, while all the other marks appear fewer than three times.

33. Answer: B

By the rules of vector algebra (Section 5.14), if $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$,

$$\begin{aligned} 6\mathbf{b} + 2\mathbf{a} &= 6\begin{pmatrix} -2 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 3 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 6(-2) \\ 6(1) \end{pmatrix} + \begin{pmatrix} 2(3) \\ 2(1) \end{pmatrix} \\ &= \begin{pmatrix} -12 \\ 6 \end{pmatrix} + \begin{pmatrix} 6 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} -6 \\ 8 \end{pmatrix} \end{aligned}$$

34. Answer: C

Substituting $p = 2$ and $q = \frac{3}{4}$ into the expression $p^2(q-1)$, we get

$$\begin{aligned} 2^2\left(\frac{3}{4} - 1\right) &= 4\left(\frac{3}{4} - \frac{4}{4}\right) \\ &= 4\left(-\frac{1}{4}\right) \\ &= -1 \end{aligned}$$

35. Answer: C

If Ivy and Abbey share an amount of GH¢ 30.00 in the ratio 3 : 2, Abbey's share is

$$\begin{aligned} \frac{2}{3+2} \text{ of GH¢ } 30 &= \frac{2}{5} \text{ of GH¢ } 30 \\ &= \frac{2}{5} \times \text{GH¢ } 30 \\ &= \text{GH¢ } 12.00 \end{aligned}$$

36. Answer: B

Mark	0	1	2	3	4	5
Frequency	1	2	7	5	4	3

The table gives the marks a certain number of learners scored on a test.

From the table, the number of learners who took the test was

$$1 + 2 + 7 + 5 + 4 + 3 = 22$$

This is the sum of the frequencies given in the table.

To find the median, we can imagine listing all the 22 marks in ascending order and finding the score in the middle.

Doing so will give the 11th and the 12th marks as the two marks in the middle. So, we need to find the 11th and 12th marks.

When the marks the learners scored are arranged in ascending order, the mark in the 1st position is 0. Since only one person scored 0, the mark in the 2nd position is 1. Since two people scored 1, the mark in the 3rd position is also 1. Continuing in this way, we get the table below:

Mark	Frequency	Positions
0	1	1
1	2	2–3
2	7	4–10
3	5	11–15
4	4	16–19
5	3	20–22

From the table, the marks in both the 11th and the 12th positions are 3.

When there are two marks in the middle, the median is their average. Therefore, the median is

$$\frac{3+3}{2} = \frac{6}{2} = 3$$

37. Answer: C

Mark	0	1	2	3	4	5
Frequency	1	2	7	5	4	3

The table gives the marks a certain number of learners scored on a test.

The probability that a learner selected at random scored 2 is given by

$$\frac{\text{the number of learners who scored 2}}{\text{the total number of learners}}$$

The number of learners who scored 2 is 7, while the total number of learners who took the test is

$$1 + 2 + 7 + 5 + 4 + 3 = 22$$

This is the sum of the frequencies given in the table.

Hence, the probability that a learner selected at random scored 2 is

$$\frac{7}{22}$$

38. Answer: C

To solve the equation $\frac{3}{4}x = 2 + \frac{1}{4}$, we can simplify it by multiplying through by 4 to get rid of the fractions.

$$\begin{aligned}\frac{3}{4}x &= 2 + \frac{1}{4}\\ 4 \times \frac{3}{4}x &= 4 \times 2 + 4 \times \frac{1}{4}\\ 3x &= 8 + 1\\ 3x &= 9\\ \frac{3x}{3} &= \frac{9}{3}\\ x &= 3\end{aligned}$$

39. Answer: D

If the product of three numbers is 90 and two of the numbers are 6 and 3, we can find the third number by dividing 90 by the product of the two numbers, 6 and 3.

Doing so, we find the third number to be

$$\frac{90}{6 \times 3} = 5$$

40. Answer: D

We simplify $x - 5(3 - 2x) - 12x + 7$ by expanding brackets and grouping like terms.

$$\begin{aligned}x - 5(3 - 2x) - 12x + 7 &= x - 15 + 10x - 12x + 7\\ &= x + 10x - 12x - 15 + 7\\ &= -x - 8\end{aligned}$$

Chapter 3

2026 Paper 2

1. (a) Given that $K = \{1, 2, 3, \ldots, 11\}$:

(i) list the prime numbers in K;

(ii) find the probability that a number selected at random from the set K is **not** a prime number.

(b) Factorize completely: $(x + y)(2m - n) - m(x + y)$.

(c) A typist charges GH¢ 68.00 for the first 8 sheets typed, and GH¢ 11.00 for each additional sheet. Calculate the amount earned if the typist typed a total of 35 sheets.

2. (a) The image of $P(2, 5)$ when translated by the vector **r** is $(-3, 8)$. Find:

(i) **r**;

(ii) the image Q' of $Q(-4, -6)$ when translated by **r**.

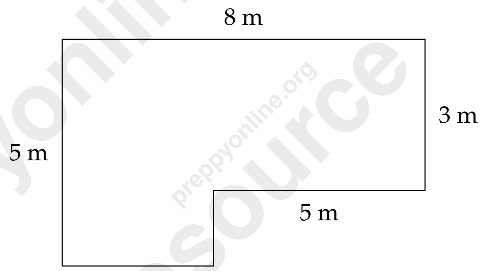
(b) Joyce and Richard contributed GH¢ 10,000.00 and GH¢ 5,000.00 respectively to start a business. They agreed that Richard will be paid **one-third** of the profit as a manager and the rest of the profit will be shared in the ratio of their contributions. If the profit of GH¢ 9,000.00 was made, how much did:

(i) Joyce receive;

(ii) Richard receive in total?

3. (a) Asana was engaged to collect cashew-fruits on a farm and was paid a wage of GH¢ 25.00 a day. If the daily wage increased by 10% and she worked for 30 days, how much will she be paid?

(b)



NOT DRAWN TO SCALE

The diagram shows the floor of a room with its dimensions. Find the:

(i) perimeter;

(ii) area;

(iii) cost of carpeting the floor if a carpet costs GH¢ 20.00 per square metre.

4. (a) In a school, the monthly income of three workers Aku, Brako and Dagadu are GH¢ 5,000.00, GH¢ 6,500.00 and GH¢ 4,200.00 respectively.

(i) Calculate the yearly income of **each** worker.

(ii) Find the yearly income difference **between** Aku and Dagadu.

(iii) Find the total yearly income of **all** the three workers.

(b) A painter places a ladder against a building at the window. The angle the foot of the ladder makes with the horizontal ground is 60°. If the distance from the foot of the ladder to the base of the building is 5 m:

(i) illustrate the information in a diagram;

(ii) find, correct to one decimal place, the:

(α) length of the ladder;

(β) distance between the window and the foot of the building.

[Take $\tan 60° = 1.732$, and $\cos 60° = \frac{1}{2}$]

5. (a) Simplify $\frac{\sqrt{72}}{\sqrt{18}-\sqrt{12}}$, leave the answer in the form $a+b\sqrt{c}$, where a, b and c are integers.

(b) Solve: $\frac{1}{2}(2x+1) \geqslant \frac{1}{3}x + 1\frac{9}{10}$.

(c) An athlete runs four times round a circular track of radius 70 m. Find, in metres, the total distance covered by the athlete. [Take $\pi = \frac{22}{7}$]

6. The data shows the shoe sizes of learners in a school.

6	4	7	5	5	6
4	5	5	4	6	7
5	6	4	6	7	5
6	4	7	5	4	6
5	5	4	6	7	5

(a) Construct a frequency distribution table for the data.

(b) If the school supplies shoes to the learners:

(i) which size should be purchased in large quantities?

(ii) give reason for the answer in (b)(i);

(iii) which size will be purchased in less quantities?

(iv) give reason for the answer in (b)(iii).

(c) Find, correct to the **nearest** whole number, the mean shoe size.

Chapter 4

Solutions to 2026 Paper 2

Question 1

(a) Given that $K = \{1, 2, 3, \ldots, 11\}$:

(i) A prime number is a natural number greater than 1 whose only factors are 1 and itself (Section 5.2.2)

Looking at the elements in

$$K = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$$

the prime numbers are

$$\{2, 3, 5, 7, 11\}$$

4, 6, 8, 9, and 10 are not prime numbers as they have factors other than 1 and themselves. 1 is also not prime as prime numbers must be greater than 1.

(ii) The probability that a number selected at random from the set K is not a prime number is

$$\frac{\text{the number of non-prime elements in } K}{\text{the number of elements in } K}$$

The numbers in K that are not prime are

$$\{1, 4, 6, 8, 9, 10\}$$

Thus, there are 6 of them.

Since there are 11 elements in K, the probability that a number chosen at random is not prime is

$$\frac{6}{11}$$

(b) Both of the terms in the expression $(x+y)(2m-n) - m(x+y)$ have the factor $(x+y)$ in common so we can factor that out to get

$$\begin{aligned} &(x+y)(2m-n) - m(x+y) \\ &= (x+y)\left((2m-n) - m\right) \\ &= (x+y)(2m-n-m) \\ &= (x+y)(m-n) \end{aligned}$$

(c) The typist charges GH¢ 68.00 for the first 8 sheets typed, and GH¢ 11.00 for each additional sheet.

Since she typed 35 sheets in total, after typing the first 8 sheets, she typed an additional

$$35 - 8 = 27 \text{ sheets}$$

Therefore, the total amount she earned was

$$\begin{aligned} &\text{the charge for the first 8 sheets} \\ &+ \text{the charge for the additional 27 sheets} \\ &= \text{GH¢ } 68.00 + 27 \times \text{GH¢ } 11.00 \end{aligned}$$

This simplifies to

$$\text{GH¢ } 68.00 + \text{GH¢ } 297.00 = \text{GH¢ } 365.00$$

Question 2

(a) (i) When a point is translated by a vector, its image is obtained by adding the translation vector to the position vector of the point.

Therefore, in this case, the translation of $P(2, 5)$ by $\mathbf{r}$ into its image $(-3, 8)$ can be written as

$$\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \mathbf{r} = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$$

By the rules of vector algebra (Section 5.14.1), we can subtract $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ from both sides to get

$$\mathbf{r} = \begin{pmatrix} -3 \\ 8 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -3-2 \\ 8-5 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

(ii) The image of $Q(-4, -6)$ when translated by $\mathbf{r}$ is given by

$$\begin{pmatrix} -4 \\ -6 \end{pmatrix} + \mathbf{r}$$

Substituting $\mathbf{r} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$ into the expression, we get

$$\begin{pmatrix} -4 \\ -6 \end{pmatrix} + \begin{pmatrix} -5 \\ 3 \end{pmatrix} = \begin{pmatrix} -4-5 \\ -6+3 \end{pmatrix} = \begin{pmatrix} -9 \\ -3 \end{pmatrix}$$

Therefore the image of $Q(-4, -6)$ when translated by $\mathbf{r}$ is $Q'(-9, -3)$.

(b) (i) Joyce and Richard agreed that whatever profit was made, Richard would get $\frac{1}{3}$ of it. They also agreed that after Richard taking $\frac{1}{3}$ of the profit, the portion remaining would be shared in the ratio of their contributions. After Richard took $\frac{1}{3}$ of the profit, the fraction remaining was

$$1 - \frac{1}{3} = \frac{3}{3} - \frac{1}{3} = \frac{2}{3}$$

It was this $\frac{2}{3}$ of the profit that was to be shared in the ratio of their contributions. The ratio of their contributions was

$$\begin{aligned} &\text{Joyce's contrib.} : \text{Richard's contrib.} \\ &= 10,000 : 5,000 \\ &= 2 : 1 \end{aligned}$$

Therefore, of the remaining $\frac{2}{3}$ of the profit, Joyce got

$$\frac{2}{1+2} = \frac{2}{3}$$

Since the profit was GH¢ 9,000.00, the portion Joyce got was

$$\begin{aligned} \frac{2}{3} \times \frac{2}{3} \times \text{GH¢ } 9,000 &= \frac{4}{9} \times \text{GH¢ } 9,000 \\ &= \text{GH¢ } 4,000.00 \end{aligned}$$

(ii) To find the amount Richard received in total, we first note the $\frac{1}{3}$ of the profit he got plus the additional amount he got from the remaining $\frac{2}{3}$ of the profit that was shared in the ratio of their contributions.

From the first $\frac{1}{3}$, Richard got

$$\frac{1}{3} \times \text{GH¢ } 9,000 = \text{GH¢ } 3,000.00$$

From the remaining $\frac{2}{3}$, he got

$$\frac{1}{1+2} = \frac{1}{3}$$

This amounted to

$$\begin{aligned} \frac{2}{3} \times \frac{1}{3} \times \text{GH¢ } 9,000 &= \frac{2}{9} \times \text{GH¢ } 9,000 \\ &= \text{GH¢ } 2,000.00 \end{aligned}$$

Hence, the total amount Richard received was

$$\text{GH¢ } 3,000 + \text{GH¢ } 2,000 = \text{GH¢ } 5,000.00$$

Question 3

(a) The daily wage Asana was to be paid was GH¢ 25.00. However, this wage was increased by 10%. After the 10% increase, the daily wage became

110% of GH¢ 25.00

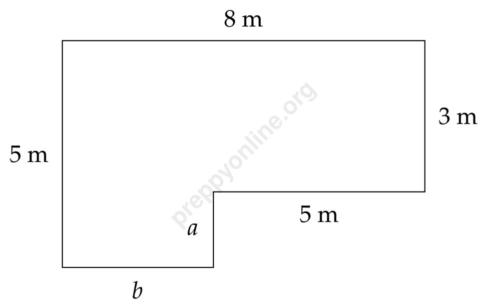
This evaluates to

$$\frac{110}{100} \times \text{GH¢ } 25 = \text{GH¢ } 275.00$$

Since she worked for 30 days with a daily wage of GH¢ 275.00, the amount she was to be paid was

$$30 \times \text{GH¢ } 275.00 = \text{GH¢ } 8,250.00$$

(b) (i) To find the perimeter of the floor, we need to find the distance around the entire figure. To do that, we can find a and b and add them to the distances or lengths already given.



From the lengths given, we have

$$3 \text{ m} + a = 5 \text{ m}$$

Therefore, $a = 2$ m.

Similarly,

$$b + 5 \text{ m} = 8 \text{ m}$$

Therefore $b = 3$ m.

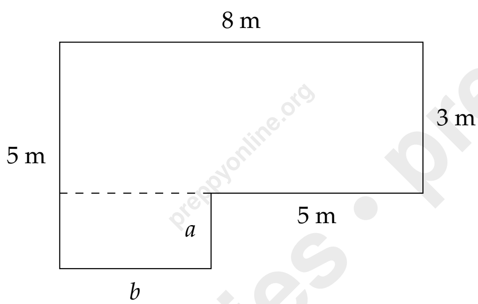
Starting from the top of the figure and moving clockwise, the perimeter (which is the sum of the lengths of all the sides) is

$$(8 + 3 + 5 + a + b + 5) \text{ m}$$

Substituting $a = 2$ and $b = 3$, into the expression, we get the perimeter as

$$\begin{aligned} &(8 + 3 + 5 + a + b + 5) \text{ m} \\ &= (8 + 3 + 5 + 2 + 3 + 5) \text{ m} \\ &= 26 \text{ m} \end{aligned}$$

(ii) To find the area of the floor, we may divide it up into smaller parts, find the area of each smaller part, and add them up.



We've now divided the floor into two rectangles: a bigger rectangle and a smaller rectangle.

The dimensions of the bigger rectangle are 8 m by 3 m while the dimensions of the smaller rectangle are $a = 2$ m by $b = 3$ m.

Hence, the area of the bigger rectangle is

$$8 \text{ m} \times 3 \text{ m} = 24 \text{ m}^2$$

And the area of the smaller rectangle is

$$2 \text{ m} \times 3 \text{ m} = 6 \text{ m}^2$$

Therefore, the area of the entire floor is

$$24 \text{ m}^2 + 6 \text{ m}^2 = 30 \text{ m}^2$$

(iii) Since the area of the floor is 30 m^2 and the carpet costs GH¢ 20.00 per square metre, the cost of carpeting the floor is

$$30 \times \text{GH¢ } 20.00 = \text{GH¢ } 600.00$$

Question 4

(a) (i) Since 1 year has 12 months, we can get the yearly income of each worker by multiplying their monthly income by 12.

Thus, since Aku's monthly income is GH¢ 5,000.00, her yearly income is

$$12 \times \text{GH¢ } 5,000.00 = \text{GH¢ } 60,000.00$$

Similarly, since Brako's monthly income is GH¢ 6,500.00, his yearly income is

$$12 \times \text{GH¢ } 6,500.00 = \text{GH¢ } 78,000.00$$

And since Dagadu's monthly income is GH¢ 4,200.00, his yearly income is

$$12 \times \text{GH¢ } 4,200.00 = \text{GH¢ } 50,400.00$$

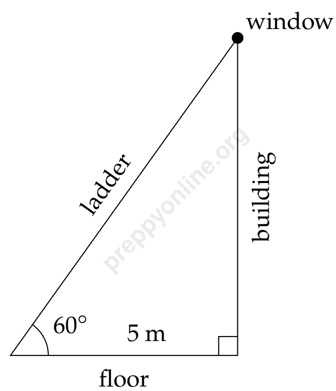
(ii) Since Aku's yearly income is GH¢ 60,000.00 and Dagadu's yearly income is GH¢ 50,400.00, the difference between their yearly incomes is

$$\text{GH¢ } 60,000 - \text{GH¢ } 50,400 = \text{GH¢ } 9,600.00$$

(iii) The total yearly income of all the three workers is

$$\begin{aligned} &\text{Aku's yearly income} \\ &+ \text{Brako's yearly income} \\ &+ \text{Dagadu's yearly income} \\ &= \text{GH¢ } 60,000 + \text{GH¢ } 78,000 + \text{GH¢ } 50,400 \\ &= \text{GH¢ } 188,400.00 \end{aligned}$$

(b) (i) We can illustrate the given information with the diagram below.



(ii) (α) From the diagram,

$$\cos 60^\circ = \frac{5\text{ m}}{\text{the length of the ladder}}$$

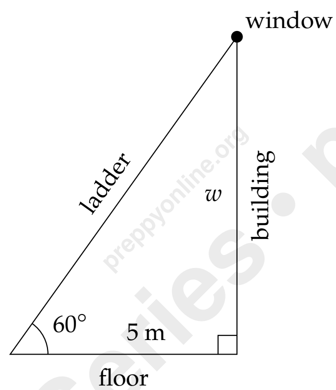
Since $\cos 60^\circ = \frac{1}{2}$, we have

$$\frac{1}{2} = \frac{5\text{ m}}{\text{the length of the ladder}}$$

Cross-multiplying and simplifying gives

$$\begin{aligned}&\text{the length of the ladder}\\ &= 2 \times 5\text{ m}\\ &= 10.0\text{ m (to one decimal place)}\end{aligned}$$

(β) The distance between the window and the foot of the building is w in the diagram below.



Thus,

$$\tan 60^\circ = \frac{w}{5\text{ m}}$$

Since $\tan 60^\circ = 1.732$, we have

$$1.732 = \frac{w}{5\text{ m}}$$

Solving for w gives

$$\begin{aligned}w &= 1.732 \times 5\text{ m}\\ &= 8.66\text{ m}\\ &= 8.7\text{ m (to one decimal place)}\end{aligned}$$

Question 5

(a) To simplify $\dfrac{\sqrt{72}}{\sqrt{18}-\sqrt{12}}$, we apply the rules of surds (Section 5.10).

Whenever possible, we will try to write the number under the square root as a product of a perfect square and another number.

For example, 72 can be written as the product of the perfect square 36 and the number 2:

$$72 = 36 \times 2$$

18 can be written as the product of the perfect square 9 and the number 2:

$$18 = 9 \times 2$$

and 12 can be written as the product of the perfect square 4 and the number 3:

$$12 = 4 \times 3$$

Using these three facts, we can simplify the given expression as

$$\begin{aligned}\frac{\sqrt{72}}{\sqrt{18}-\sqrt{12}} &= \frac{\sqrt{36\times 2}}{\sqrt{9\times 2}-\sqrt{4\times 3}}\\ &= \frac{\sqrt{36}\sqrt{2}}{\sqrt{9}\sqrt{2}-\sqrt{4}\sqrt{3}}\\ &= \frac{6\sqrt{2}}{3\sqrt{2}-2\sqrt{3}}\end{aligned}$$

Because we have been asked to leave the answer in the form $a + b\sqrt{c}$, where a, b and c are integers, we have to rationalize the denominator (Section 5.10.3) of the expression above. This means multiplying both the numerator and the denominator of the expression by the conjugate surd of the denominator.

Since the conjugate surd of the denominator is $3\sqrt{2}+2\sqrt{3}$, we get

$$\begin{aligned}&\frac{6\sqrt{2}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\\ &= \frac{6\sqrt{2}\left(3\sqrt{2}+2\sqrt{3}\right)}{\left(3\sqrt{2}-2\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)}\\ &= \frac{18\sqrt{2}\sqrt{2}+12\sqrt{2}\sqrt{3}}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}\\ &= \frac{18(2)+12\sqrt{6}}{9(2)-4(3)}\\ &= \frac{36+12\sqrt{6}}{18-12}\\ &= \frac{36+12\sqrt{6}}{6}\\ &= 6+2\sqrt{6}\end{aligned}$$

(b) To solve for x in $\frac{1}{2}(2x+1) \geqslant \frac{1}{3}x + 1\frac{9}{10}$, we first convert $1\frac{9}{10}$ into an improper fraction:

$$1\frac{9}{10} = \frac{19}{10}$$

So, the inequality becomes

$$\frac{1}{2}(2x+1) \geqslant \frac{1}{3}x + \frac{19}{10}$$

At this point, we have a number of ways to proceed. One way is to multiply both sides of the inequality by a number (like the LCM of the denominators or simply their product) to clear the fractions. Another way is to first expand the brackets and simplify, working with the fractions.

We shall use both methods.

Method 1

We multiply both sides of the inequality by the product of the denominators of the fractions, 2(3)(10), and simplify.

$$\begin{aligned}
\frac{1}{2}(2x+1) &\geqslant \frac{1}{3}x + \frac{19}{10} \\
2(3)(10) \times \frac{1}{2}(2x+1) &\geqslant 2(3)(10) \times \frac{1}{3}x + 2(3)(10) \times \frac{19}{10} \\
3(10)(2x+1) &\geqslant 2(10)x + 2(3)(19) \\
30(2x+1) &\geqslant 20x + 114 \\
60x + 30 &\geqslant 20x + 114 \\
60x - 20x &\geqslant 114 - 30 \\
40x &\geqslant 84 \\
x &\geqslant \frac{84}{40} \\
x &\geqslant \frac{21}{10} \\
x &\geqslant 2\frac{1}{10}
\end{aligned}$$

Method 2

In this method, we use the fractions to expand the brackets and simplify the resulting inequality.

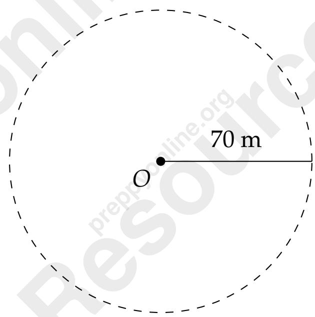
$$\begin{aligned}
\frac{1}{2}(2x+1) &\geqslant \frac{1}{3}x + \frac{19}{10} \\
x + \frac{1}{2} &\geqslant \frac{1}{3}x + \frac{19}{10} \\
x - \frac{1}{3}x &\geqslant \frac{19}{10} - \frac{1}{2} \\
\frac{3}{3}x - \frac{1}{3}x &\geqslant \frac{19}{10} - \frac{5}{10} \\
\frac{2}{3}x &\geqslant \frac{14}{10}
\end{aligned}$$

To isolate x, we multiply both sides of the inequality by the reciprocal of $\frac{2}{3}$:

$$\begin{aligned}
\frac{2}{3}x &\geqslant \frac{14}{10} \\
\frac{3}{2} \times \frac{2}{3}x &\geqslant \frac{3}{2} \times \frac{14}{10} \\
x &\geqslant \frac{21}{10}
\end{aligned}$$

(c) Below is a sketch of the circular track.

Its centre O and radius of 70 metres are labelled.



When the athlete runs round the track, the path he traces is the dashed line—the circumference of the circle.

Since the circumference of a circle is given by $2\pi r$, whenever the athlete runs round the circular track, he covers a distance of

$$2 \times \pi \times 70 \text{ m} = 2 \times \frac{22}{7} \times 70 \text{ m} = 440 \text{ m}$$

Therefore, since he run four times around the track, the total distance he covered was

$$4 \times 440 \text{ m} = 1,760 \text{ m}$$

Question 6

(a) Below is a frequency distribution table for the data.

Shoe Size (x)	Tally	Frequency (f)
4	卌 ||	7
5	卌 卌	10
6	卌 |||	8
7	卌	5

(b) (i) The shoe size to be purchased in the largest quantities should be size 5.

(ii) Shoe size 5 should be the one purchased in the largest quantities because it is the size with the highest number of learners wearing it.

(iii) The shoe size to be purchased in the smallest quantities should be size 7.

(iv) Shoe size 7 should be the one purchased in the smallest quantities because it is the size with the lowest number of learners wearing it.

(c) To find the mean shoe size, we extend the frequency distribution table to add a column, fx, for the product of shoe size (x) and frequency (f).

Then the mean shoe size will be the total in the fx column divided by the total in the frequency column. That is,

$$\frac{\sum fx}{\sum f}$$

Shoe Size (x)	Frequency (f)	fx
4	7	28
5	10	50
6	8	48
7	5	35
	$\sum f = 30$	$\sum fx = 161$

From the table, the mean shoe size is

$$\frac{\sum fx}{\sum f} = \frac{161}{30}$$
$$= 5\frac{11}{30}$$
$$= 5 \text{ (to the nearest whole number)}$$

Chapter 5

Quick Revision Notes

5.1 Sets

A set is a well-defined collection of unique objects.

An example of a set is the set of counting numbers less than 5: $\{1, 2, 3, 4\}$.

Another example is the set of colors in a rainbow: {red, orange, yellow, green, blue, indigo, violet}.

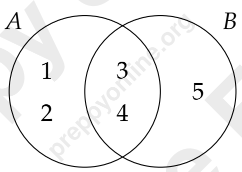
Note that the elements of a set must be unique. That means we can't have repeated elements in a set. For example, $\{1, 2, 2, 3\}$ is not set because 2 is repeated.

Sets are often denoted by capital letters, e.g., $E = \{2, 4, 6, 8\}$.

5.1.1 Venn diagrams

Venn diagrams are a useful tool for showing the relationships between sets.

If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$, we can represent the relationship between A and B with the diagram below.



Notice that because the elements 3 and 4 are in both sets, they are shown in the part of the diagram where the two circles that represent the two sets overlap.

5.1.2 Types of sets

Finite sets

A finite set is a set with a countable number of elements. That means its number of elements can be represented by a non-negative integer such as 0, 1, 2, 3, and so on.

An example of a finite set is the set of vowels in the English alphabet: {a, e, i, o, u}.

Infinite sets

An infinite set is a set with an unlimited number of elements. That means its number of elements cannot be represented by a natural number.

An example of an infinite set is the set of whole numbers, $\{0, 1, 2, 3, 4, 5, \ldots\}$. This set is infinite because there is no such thing as the largest whole number. Hence, the set is endless.

The empty set

The empty set is the set with no elements. That means its number of elements is 0.

They empty set is denoted by empty curly braces, $\{\}$. It is also denoted by $\varnothing$.

Singletons

A singleton is a set with exactly one element. That means its number of elements is 1.

Examples of singleton sets are the following

$$\{8\} \quad \{\text{A}\} \quad \{0\} \quad \{\text{orange}\}$$

Is the set $\{\{1, 2, 3\}\}$ a singleton?

Yes, it is. It has only one element, the set $\{1, 2, 3\}$.

5.1.3 Subsets

Given a set A, a subset of A is a set whose elements are also elements of A.

For instance, the set $\{3, 4\}$ is a subset of $\{1, 2, 3, 4, 5\}$ because each of its elements—3 and 4—is an element of the set $\{1, 2, 3, 4, 5\}$.

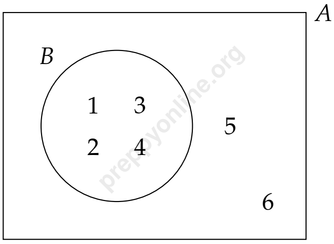
Is $\{1, 2, 3, 4, 5\}$ a subset of $\{1, 2, 3, 4, 5\}$?

Yes, it is. Using the definition above, we have to check if every element of $\{1, 2, 3, 4, 5\}$ is an element of $\{1, 2, 3, 4, 5\}$. As this is obviously the case, $\{1, 2, 3, 4, 5\}$ is a subset of itself.

By this reasoning, every set is a subset of itself.

We write $A \subseteq B$ to denote that A is a subset of B.

The diagram below shows set A, represented by the rectangle, and set B represented by the circle. From the diagram, set B is a subset of set A because every element in B is also in A.



The elements of B are 1, 2, 3, 4, while the elements of A are 1, 2, 3, 4, 5, 6.

Proper subsets

Sometimes we only want subsets of a set that are not the set itself. These are called proper subsets. Hence, though every set is a subset of itself, it is not a proper subset of itself.

$\{3, 4, 5\}$ is a proper subset of $\{1, 2, 3, 4, 5\}$ but $\{1, 2, 3, 4, 5\}$ is not a proper subset of $\{1, 2, 3, 4, 5\}$.

5.1.4 The number of subsets of a set

The number of subsets of a set with n elements is 2^n.

For example, the set $\{1, 2, 3\}$ has $2^3 = 8$ subsets because it has 3 elements. The 8 subsets are the following:

$$\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$$

Notice that the list of subsets includes the empty set, $\{\}$, and the set itself $\{1, 2, 3\}$. The empty set is a subset of every set. Every set is also a subset of itself.

Since $\{1, 2, 3, 4, 5\}$ has 5 elements, it has $2^5 = 32$ subsets. Can you list them?

5.1.5 Comparing sets

Equal sets

Two sets are equal if they have the same elements. For example, $\{1, 2, 3\}$ and $\{3, 2, 1\}$ are equal because they have the same elements. This means the order in which the elements of a set are arranged does not matter. Once two sets have the same elements, they are equal.

Equivalent sets

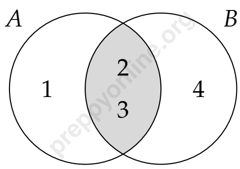
Two sets are equivalent if they have the same number of elements. For example, $\{1, 2, 3\}$ and $\{4, 5, 6\}$ are equivalent because they have the same number of elements, 3.

Disjoint sets

Two sets are called disjoint if they have no elements in common. For example, $A = \{1, 2\}$ and $B = \{3, 4, 5\}$ are disjoint because they have no elements in common.

Intersecting sets

If two sets are not disjoint, then they are intersecting sets. Intersecting sets are sets that have elements in common. For example, $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$ are intersecting sets because they have 2 and 3 in common. Their intersection is shaded in the Venn diagram below.



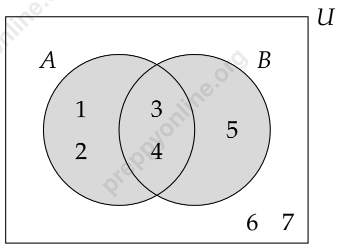
5.1.6 Operations on sets

Union

The union of sets A and B, denoted $A \cup B$, is the set that contains all the elements of both sets.

For example, the union of $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$ is $A \cup B = \{1, 2, 3, 4\}$.

Let A and B be subsets of the universal set $U = \{1, 2, 3, 4, 5, 6, 7\}$. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$, then $A \cup B = \{1, 2, 3, 4, 5\}$ and the relationships between the sets can be represented by the Venn diagram below. The shaded region represents the union.

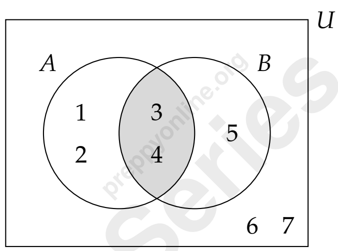


Intersection

The intersection of sets A and B, denoted $A \cap B$, is the set that contains all the elements that are in both sets.

For example, the intersection of $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$ is $A \cap B = \{2, 3\}$.

Let A and B be subsets of the universal set $U = \{1, 2, 3, 4, 5, 6, 7\}$. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$, then $A \cap B = \{3, 4\}$ and the relationships between the sets can be represented by the Venn diagram below. The shaded region represents the intersection.

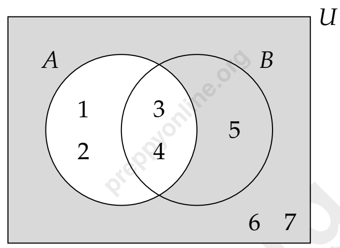


Complement

The complement of a set is the set that contains all the elements that are not in the set. For example, the complement of $A = \{1, 2, 3\}$ in $\{1, 2, 3, 4, 5\}$ is $A' = \{4, 5\}$. The complement of A is denoted A'.

Whenever we talk of complements, we must do so in reference to a larger set. For example, the complement of $\{1, 2, 3\}$ in $\{1, 2, 3, 4, 5\}$ is $\{4, 5\}$. But the complement of $\{1, 2, 3\}$ in $\{1, 2, 3, 4, 5, 6\}$ is $\{4, 5, 6\}$.

Let A and B be subsets of the universal set $U = \{1, 2, 3, 4, 5, 6, 7\}$. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$, then $A' = \{5, 6, 7\}$ and the relationships between the sets can be represented by the Venn diagram below. The shaded region represents A', the complement of A.



5.2 Real numbers

The set of real numbers is, roughly speaking, the set of all numbers that can be represented on a number line. It comprises the set of rational numbers (positive and negative integers, zero, and fractions) and the set of irrational numbers (non-repeating decimals like π and $\sqrt{2}$).

5.2.1 Properties of arithmetic operations on real numbers

The commutative property

An operation is said to be commutative if the order of the operands does not change the result.

For example, addition is commutative because the order in which numbers are added in a sum does not change the answer.

For example, $2 + 3 = 3 + 2$.

Similarly, $2 + 3 + 4 = 4 + 2 + 3 = 3 + 2 + 4$.

Subtraction is not commutative. This means that when subtracting numbers, order matters. Changing the order of the numbers will give a different answer.

For example, $2 - 3 \neq 3 - 2$.

Multiplication is commutative. Again, this means that when multiplying numbers, we may do so in any order.

For example, $2 \times 3 = 3 \times 2$.

Similarly, $2 \times 3 \times 4 = 4 \times 2 \times 3 = 3 \times 2 \times 4$.

Division is not commutative. Order is important when dividing. $4 \div 2 \neq 2 \div 4$.

The associative property

This property is about the grouping of numbers when doing arithmetic with three or more numbers. If changing the grouping of the operands does not change the result, the operation is associative.

For example, when adding three numbers, it does not matter how we group the numbers with parentheses:

$$(1 + 2) + 3 = 1 + (2 + 3)$$

This is because addition of numbers is associative.

Multiplication is also associative. For example, when multiplying three numbers, it does not matter how we group them:

$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

Subtraction and division are, however, not associative. Changing the grouping of the numbers changes the result. For example,

$$(1 - 2) - 3 = -4 \quad \text{but} \quad 1 - (2 - 3) = 2$$

The distributive property

The distributive property of multiplication over addition is the property that when multiplying a sum by a number, we can first take the sum and multiply by the number or multiply the number by each of the addends and then take the sum of these new products.

For example,

$$2 \times (3 + 4) = 2 \times 7 \quad \text{and also} \quad 2 \times (3 + 4) = 2 \times 3 + 2 \times 4$$

5.2.2 Prime factorization

A prime number is a natural number greater than 1 whose only factors are 1 and itself.

Prime factorization is the process of breaking up a composite number into a product of prime numbers.

Every integer greater than 1 is either prime or the product of a collection of primes.

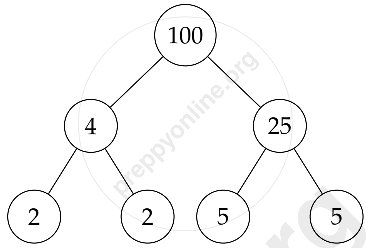
For example,

- 5 is prime.
- 15 is composite. It is the product of 3 and 5.
- 100 is composite. It is the product of the primes 2, 2, 5, and 5.
- 13 is prime.

To break an integer into a product of primes, we may first break it up into a product of any two integers and then break up those integers into a product of integers until we get a product of primes. This process can be illustrated with a factor tree.

For example, $100 = 4 \times 25$. Both 4 and 25 are not prime but can be written as products of primes: $4 = 2 \times 2$ and $25 = 5 \times 5$. Thus, $100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$.

This is what the factor tree below shows.



5.2.3 Standard form

To write a number in standard form, we need to write it in the form $a \times 10^n$ where $1 \leqslant a < 10$ and n is an integer.

For example, to write 0.00459 in standard form, we need to keep moving the decimal point to the right until it is after the first non-zero number. Doing that while keeping track of the number of rimes we moved, we get

$$0.00459 = 4.59 \times 10^{-3}$$

5.3 Percentages

A percentage is a fraction of 100.

For example,

$$50\% = \frac{50}{100}$$

To express a number as a percentage, we multiply by 100%.

For example, $\frac{1}{4}$ can be expressed as a percentage as

$$\frac{1}{4} \times 100\% = 25\%$$

5.4 Simple interest

The formula for simple interest is

$$\text{simple interest} = \text{principal} \times \text{rate} \times \text{time}$$

The principal is the amount borrowed or invested.

The rate is how much the principal increases per given period. It can be stated per annum (same as per year), per month, per week, etc.

The time is how long the amount is borrowed or invested.

For example, to find the simple interest on GH¢ 2,000.00 invested at 10% simple interest per annum for a period of 9 months, we extract the relevant information.

The principal is GH¢ 2,000.00.

The rate is 10% per annum or per year.

The time is 9 months. However, because the rate is given in terms of years, we would like to convert the time also to years.

Since there are 12 months in a year, 9 months is equivalent to $\frac{9}{12}$ years.

With these three values, the simple interest may be computed as

$$\begin{aligned} \text{simple interest} &= \text{GH¢ } 2000 \times 10\% \times \frac{9}{12} \\ &= 2000 \times \frac{10}{100} \times \frac{9}{12} \\ &= \text{GH¢ } 150.00 \end{aligned}$$

5.5 Ratio and proportion

5.5.1 Ratios

A ratio is a way of expressing the relationship between two numbers.

For example, if there are 4 mangoes and 6 oranges in a basket, the ratio of mangoes to oranges is 4 : 6.

This ratio may also be written as $\frac{4}{6}$. Hence, ratios and fractions can be used to represent the same information.

Just as the fraction $\frac{4}{6}$ is equivalent to $\frac{2}{3}$, the ratio 4 : 6 is equivalent to 2 : 3.

Ratios tell us how a number is divided and the relative sizes of the portions.

For example, if the ratio of red pens to blue pens in a box is 2 : 3 and there is a total of 15 pens in the box, how many are red?

The ratio 2 : 3 tells us that for every 2 red pens there are, there are 3 blue pens.

Thus, if we see 4 red pens, we should see 6 blue pens; if we see 6 red pens, there should be 9 blue pens, and so on.

The fraction of pens in the box that are red is

$$\frac{2}{2+3} = \frac{2}{5}$$

Hence, the number of red pens is

$$\frac{2}{5} \times 15 = 6$$

5.5.2 Proportions

We get a proportion when we set two ratios equal to each other.

For example, if the ratio of boys to girls in a class is 3 : 4, and there are 15 boys in the class, how many girls are there?

This information can be presented as

$$\begin{gathered} \text{boys : girls} \\ 3 : 4 \\ 15 : x \end{gathered}$$

Since the three ratios above are equal, we can write

$$3 : 4 = 15 : x$$

When we do that, we get a proportion.

Since ratios can be written as fractions, we have

$$\frac{3}{4} = \frac{15}{x}$$

Cross-multiplying and solving for x gives

$$\begin{aligned} 3x &= 15(4) \\ x &= \frac{15(4)}{3} \\ x &= 20 \end{aligned}$$

5.5.3 Direct proportion

When the relationship between two quantities is such that as one increases, the other increases and when one decreases, the other decreases, there is direct relationship between them.

For example, if 10 cows eat 20 bags of food, how many bags of food will 20 cows eat?

20 cows will eat more food than 10 cows. In fact, they will eat twice as much food.

The information can be written as

$$\begin{aligned} 10 \text{ cows} &\longrightarrow 20 \text{ bags} \\ 20 \text{ cows} &\longrightarrow x \text{ bags} \end{aligned}$$

Since there is a direct relationship between the number of cows and the number of bags, we can write these equivalent fractions:

$$\frac{10}{20} = \frac{20}{x}$$

Cross-multiplying and solving for x gives

$$\begin{aligned} 10x &= 20(20) \\ x &= \frac{20(20)}{10} \\ x &= 40 \end{aligned}$$

5.5.4 Indirect or inverse proportion

When the relationship between two quantities is such that as one increases, the other decreases and when one decreases, the other increases, there is indirect or inverse relationship between them.

For example, if 5 men use 20 hours to paint a house, how many hours will 10 men use to paint it?

10 men will use less time to paint the house than 5 men will. In fact, they will use half the amount of time 5 men will use.

The information can be written as

$$5 \text{ men} \rightarrow 20 \text{ hours}$$
$$10 \text{ men} \rightarrow x \text{ hours}$$

Because of the inverse relationship between the number of men and the amount of time they will need, we can write these equivalent fractions:

$$\frac{5}{10} = \frac{x}{20}$$

Notice that because of the inverse relationship between the number of men and the number of hours, we flipped the order of the quantities on the right-hand side of the proportion.

Solving for x in the proportion above gives $x = 10$.

5.6 Factorization of algebraic expressions

5.6.1 The difference of two squares

Expressions of the form $a^2 - b^2$, where one square is subtracted from another have may be factorized as

$$a^2 - b^2 = (a - b)(a + b)$$

This is an important factorization that has many applications in mathematics and should be memorized.

The factorization may be verified by expanding it:

$$(a - b)(a + b) = a^2 + ab - ab - b^2 = a^2 - b^2$$

Using the difference of two squares factorization, $4x^2 - 9y^2$ can be factorized as

$$4x^2 - 9y^2 = (2x)^2 - (3y)^2 = (2x - 3y)(2x + 3y)$$

To evaluate $65^2 - 35^2$, we may use the difference of two squares factorization to write

$$65^2 - 35^2 = (65 - 35)(65 + 35) = 30(100) = 3000$$

5.7 Mensuration

Mensuration is all about measurement. That is, the measurement of distances, areas, volumes, etc.

5.7.1 Perimeter

The perimeter of a plane figure is the sum of the lengths of its sides or the distance around its edge.

The circumference of a circle with radius r is given by the formula

$$\text{circumference} = 2\pi r$$

The perimeter of a rectangle is given by the formula

$$\text{perimeter} = 2 \times (l + w)$$

5.7.2 Area

The area of a circle with radius r is given by the formula

$$\text{area} = \pi r^2$$

The area of a rectangle with length l and width w is given by the formula

$$\text{area} = l \times w$$

The area of a square with side length a is given by the formula

$$\text{area} = a^2$$

The area of a triangle with base b and perpendicular height h is given by the formula

$$\text{area} = \frac{1}{2} \times b \times h$$

The area of a trapezium with parallel sides a and b and perpendicular height h is given by the formula

$$\text{area} = \frac{1}{2} \times (a + b) \times h$$

The area of a parallelogram with base b and perpendicular height h is given by the formula

$$\text{area} = b \times h$$

5.7.3 Volume

The volume of a cube with side length a is given by the formula

$$\text{volume} = a^3$$

The volume of a cylinder with radius r and height h is given by the formula

$$\text{volume} = \pi r^2 h$$

The volume of a sphere with radius r is given by the formula

$$\text{volume} = \frac{4}{3}\pi r^3$$

5.8 Rules of indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

5.9 Rules of logarithms

Product rule

$$\log_b(xy) = \log_b x + \log_b y$$

For example,

$$\log_2 64 = \log_2(8 \times 8) = \log_2 8 + \log_2 8 = 3 + 3 = 6$$

Quotient rule

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

For example,

$$\log_2\left(\frac{32}{4}\right) = \log_2 32 - \log_2 4 = 5 - 2 = 3$$

Power rule

$$\log_b(x^p) = p \log_b x$$

For example,

$$\log_3 81 = \log_3 3^4 = 4 \log_3 3 = 4$$

Root rule

$$\log_b \sqrt[p]{x} = \frac{\log_b x}{p}$$

For example,

$$\begin{aligned} \log_3 \sqrt{243} &= \log_3 243^{\frac{1}{2}} \\ &= \frac{1}{2}\log_3 243 \\ &= \frac{1}{2}\log_3 3^5 \\ &= \frac{1}{2} \times 5 \\ &= \frac{5}{2} \end{aligned}$$

5.10 Surds

A surd is an irrational number expressed as a root (such as a square root or a cube root) that cannot be simplified into a whole number or exact fraction.

$\sqrt{2}$ is a surd because it cannot be simplified into a whole number or exact fraction.

$\sqrt{4}$ is not a surd because it can be simplified into the whole number 2.

$\sqrt{\frac{25}{4}}$ is not a surd because it can be simplified into the fraction $\frac{5}{2}$.

$sqrt32$ is a surd because it cannot be simplified into a whole number or exact fraction as 32 is not a perfect square.

We will restrict ourselves to square roots but the principles for other roots are similar.

5.10.1 Rules of surds

The product rule

$$\sqrt{a}\sqrt{b} = \sqrt{ab}$$

For example,

$$\sqrt{2}\sqrt{2} = \sqrt{2 \times 2} = \sqrt{4} = 2$$

Also,

$$\sqrt{5}\sqrt{2} = \sqrt{10}$$

The quotient rule

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

For example,

$$\frac{\sqrt{32}}{\sqrt{2}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4$$

5.10.2 Simplification of surds

To simplify a surd, we try to write the number under the radical as a product of a perfect square and another number.

For example, since $32 = 16 \times 2$, $\sqrt{32}$ may be written as

$$\sqrt{32} = \sqrt{16 \times 2}$$

Then, applying the rules of surds, we get

$$\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4 \times \sqrt{2} = 4\sqrt{2}$$

Similarly,

$$\sqrt{99} = \sqrt{9 \times 11} = \sqrt{9}\sqrt{11} = 3\sqrt{11}$$

5.10.3 Rationalization of the denominator

This is a technique used to eliminate surds from the denominator of a fraction. We multiply both the numerator and the denominator by a special number that the value of the fraction is not changed but the denominator becomes a rational number.

For example, $\frac{2}{\sqrt{3}}$ is the same as

$$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

When the denominator of the fraction is a sum or a difference involving a radical (for example, $a + \sqrt{b}$), we multiply by both the numerator and the denominator by the conjugate of the surd, which is the expression with the opposite sign between the two terms. That is, the conjugate of $a + \sqrt{b}$ is $a - \sqrt{b}$ and vice versa.

For example, to rationalize the denominator of $\frac{2}{2+\sqrt{3}}$, we multiply both the numerator and the denominator of the fraction by the conjugate of $2 + \sqrt{3}$:

$$\frac{2}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}$$

Using the fact that the denominator is now a difference of two squares (Section 5.6.1), we have

$$\begin{aligned}\frac{2\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)} &= \frac{2\left(2-\sqrt{3}\right)}{2^2-\left(\sqrt{3}\right)^2}\\ &= \frac{4-2\sqrt{3}}{4-3}\\ &= \frac{4-2\sqrt{3}}{1}\\ &= 4-2\sqrt{3}\end{aligned}$$

5.11 Sequences and series

Sometimes a sequence of numbers follows a pattern by which we can deduce what the other numbers in the sequence are.

For example, can you find the next number in the sequence below?

$$1, 2, 4, 7, 11, \ldots$$

If we notice the pattern below, we can guess the next number as 16.

$$1 \xrightarrow{+1} 2 \xrightarrow{+2} 4 \xrightarrow{+3} 7 \xrightarrow{+4} 11 \xrightarrow{?} \ldots$$

5.11.1 Arithmetic progressions or linear sequences

An arithmetic progression (AP) or linear sequence is a sequence of numbers in which the difference between two consecutive terms is constant. This constant difference between the terms of the sequence is called the common difference.

For example, $1, 2, 3, 4, \ldots$ is an arithmetic progression because every two consecutive terms in the sequence differ by 1. In other words, given a term of the sequence, we obtain the next term by adding 1.

$2, 4, 6, 8, \ldots$ is also an arithmetic progression because the difference between every pair of consecutive terms is constant. In particular, given a term of the sequence, the next term is obtained by adding 2.

The first term and the common difference

What is the common difference of the arithmetic progression $4, 2, 0, -2, \ldots$?

Given a term of the sequence, we must add -2 to it to get the next term. Hence, the common difference is -2.

The difference between consecutive terms of an arithmetic progression is called the common difference. It is usually denoted by d.

The first term of an arithmetic progression is usually denoted by a.

Hence, in the sequence $1, 2, 3, 4, \ldots$, the first term, $a = 1$ and the common difference, $d = 1$.

In the sequence $2, 4, 6, 8, \ldots$, the first term, $a = 2$ and the common difference, $d = 2$.

The nth term of an AP

The nth term of an arithmetic progression, denoted u_n, is given by the formula

$$u_n = a + (n - 1)d$$

For example, given the sequence $3, 5, 7, 9, \ldots$, since the first term, $a = 3$, and the common difference, $d = 2$, the 10th term, u_{10}, is given by

$$u_{10} = 3 + (10 - 1)2 = 3 + 9(2) = 3 + 18 = 21$$

Notice that we could have found the 10th term by continuing the sequence until we got to the 10th term. However, it is faster to use the formula.

5.11.2 Geometric progressions or exponential sequences

A geometric progression or exponential sequence is a sequence of numbers where successive terms are obtained by multiplying the same number, called the common ratio.

An example of a geometric progression is $1, 2, 4, 8, \ldots$. Each term in the sequence is obtained by multiplying the preceding term by the constant 2.

Another example is $81, 27, 9, 3, \ldots$. This is a geometric progression because subsequent terms are obtained by multiplying the preceding term by a constant, $\frac{1}{3}$.

The first term and the common ratio

Find the common ratio of the geometric progression $64, 32, 16, 8, \ldots$.

The common ratio is the number that we must multiply a given term by to get the next term. Given 64, the next term is 32. Since we must multiply 64 by $\frac{1}{2}$ to get 32, the common ratio is $\frac{1}{2}$.

The first term of a geometric progression is usually denoted a, while the common ratio is denoted r.

The nth term of a GP

The nth term of a geometric progression, denoted u_n, is given by the formula

$$u_n = ar^{n-1}$$

For example, since the first term of the geometric progression $1, 2, 4, 8, \ldots$ is 1 and the common ratio is 2, the 10th term of the sequence is

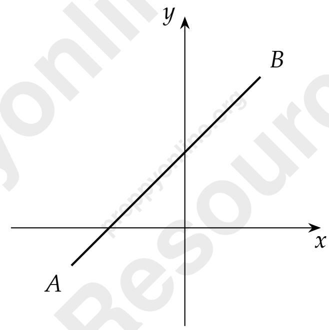
$$u_{10} = 1(2)^{10-1} = 2^9 = 512$$

5.12 Coordinate geometry

A straight line is the shortest distance between two points.

They have many applications in mathematics.

An example of a straight line is line AB below.



5.12.1 The gradient or slope of a line

Imagine climbing a mountain. The steeper the slope, the harder it is to climb; and the gentler the slope, the easier it is to climb.

The gradient or slope of a line is a measure of how steep the line is.

For example, lines A and B below have gentle slopes,

while lines C and D below have steep slopes.

To find the gradient of a line, we need to find to points (x_1, x_2) and (y_1, y_2) on the line and use the formula

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

For example, to find the gradient of the line that passes through the points $(2,5)$ and $(-1,7)$, we let one pair of coordinates be (x_1, y_1) and the other one be (x_2, y_2) and use the formula to get the gradient as

$$\frac{7-5}{-1-2} = \frac{2}{-3} = -\frac{2}{3}$$

5.12.2 The equation of a line

The equation of a line may be written in different forms.

The general form of the equation of a line

The general form of the equation of a line is

$$ax + by + c = 0$$

This means that equation of every line can be written in this form. We can't have terms like x^2, x^3, y^2, and so on in the equation of a line.

The slope-intercept form of the equation of a line

The equation of a line may be written in the form

$$y = mx + c$$

This is called the slope-intercept form of the equation of a line.

When the equation of a line is written this way, we can immediately read the slope or gradient of the line and its y-intercept. The slope is m and the y-intercept is c.

The intercept form of the equation of a line

The equation of a line may be written in the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

This is called the intercept form of the equation of a line.

When the equation of a line is written this way, the x-intercept is a and the y-intercept is b.

5.12.3 How to find the equation of a line

A line is defined by two points or a point and its gradient.

Finding the equation of a line given its gradient and a point on the line

Given the gradient m of a line and a point $P(x_1, y_1)$ on the line, we can find the equation of the line as follows.

If a general point on the line has coordinates (x, y) then we have

$$\frac{y - y_1}{x - x_1} = m \quad \text{or} \quad y - y_1 = m(x - x_1)$$

From this, we can get the equation of the line.

For example, if a line has gradient 2 and passes through the point $(3,4)$, then we can find its equation by writing

$$\begin{aligned} \frac{y-4}{x-3} &= 2 \\ y - 4 &= 2(x-3) \\ y - 4 &= 2x - 6 \\ y &= 2x - 2 \end{aligned}$$

Finding the equation of a line given two points on the line

Given two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ on a line, we can find the equation of the line by first using the two points to find the gradient. After that, we use one of the points and the gradient we found to find the equation of the line.

The gradient of the line is

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Using the point $P(x_1, y_1)$ and the gradient, we can find the equation of the line by writing

$$\begin{aligned} \frac{y - y_1}{x - x_1} &= \frac{y_2 - y_1}{x_2 - x_1} \\ y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \end{aligned}$$

For example, to find the equation of the line that passes through $(1,2)$ and $3,5$, we first find the gradient as

$$\frac{5-2}{3-1} = \frac{3}{2}$$

Using this gradient and the point $(1,2)$, we can find the equation of the line as

$$\begin{aligned}\frac{y-2}{x-1} &= \frac{3}{2}\\ y-2 &= \frac{3}{2}(x-1)\\ y-2 &= \frac{3}{2}x - \frac{3}{2}\\ y &= \frac{3}{2}x - \frac{3}{2} + 2\\ y &= \frac{3}{2}x + \frac{1}{2}\end{aligned}$$

5.12.4 The distance between two points

Given two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, the distance between them is given by the formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This distance is the same as the length of the line segment between the two points.

For example, to find the distance of the between $C(2,4)$ and $D(6,1)$, we use the formula to get

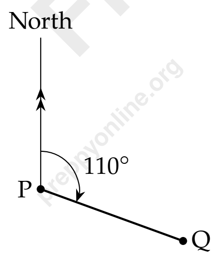
$$\begin{aligned}\sqrt{(2-6)^2 + (4-1)^2} &= \sqrt{(-4)^2 + 3^2}\\ &= \sqrt{16+9}\\ &= \sqrt{25}\\ &= 5 \text{ units}\end{aligned}$$

5.13 Bearings

Bearings are measured clockwise from due north.

For example, if the bearing or Q from P is 110°, that means that if we stand at P and start measuring clockwise from the North, Q is going to be on the line that is 110° from the North.

The figure below shows the locations of P and Q.



By convention, bearings are written with three digits. Hence, a bearing of 80° is written as 080°.

5.13.1 Back bearings

Given the bearing of Q from P, we can find the bearing of P from Q, also known as the back bearing, as follows:

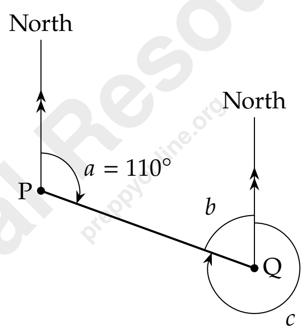
If the given bearing, x, is such that $x < 180°$, add 180° to it.

If the given bearing, x, is such that $x \geqslant 180°$, subtract 180° from it.

For example, if the bearing of Q from P is 110°, then, using the formula described above, the bearing of P from Q is $110° + 180° = 290°$.

But we could also find the back bearing by drawing a diagram and using the relationships between the angles to figure it out.

The information given can be represented with the diagram below.



The diagram shows the bearing of Q from P as angle a, while the bearing of P from Q is represented by angle c.

Because angles a and b are co-interior angles, they add up to 180°. So,

$$\begin{aligned}a + b &= 180°\\ 110° + b &= 180°\\ b &= 180° - 110°\\ b &= 70°\end{aligned}$$

Also, because angles b and c are angles at a point, we have

$$\begin{aligned}b + c &= 360°\\ 70° + c &= 360°\\ c &= 360° - 70°\\ c &= 290°\end{aligned}$$

Hence, the bearing of P from Q is 290°.

5.14 Vectors

Vectors are usually written in component form. For example, $\mathbf{v} = \begin{pmatrix} a \\ b \end{pmatrix}$. The entries in the vector are called components.

In $\mathbf{v} = \begin{pmatrix} a \\ b \end{pmatrix}$, a is called the x component while b is called the y component.

5.14.1 Rules of vector algebra

Vectors can be added, subtracted, and multiplied by numbers.

Addition of vectors

To add two vectors, we add the corresponding components.

For example, if $\mathbf{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$$\begin{aligned} \mathbf{v} + \mathbf{w} &= \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} -2+1 \\ 3+2 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 5 \end{pmatrix} \end{aligned}$$

Subtraction of vectors

Subtraction of vectors is also done component by component.

For example, if $\mathbf{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$$\begin{aligned} \mathbf{v} - \mathbf{w} &= \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} -2-1 \\ 3-2 \end{pmatrix} \\ &= \begin{pmatrix} -3 \\ 1 \end{pmatrix} \end{aligned}$$

Scalar multiplication

Vectors can be multiplied by numbers. The number by which a vector is multiplied is called a scalar.

If the vector $\mathbf{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ is multiplied by the scalar k, we get

$$k\mathbf{v} = \begin{pmatrix} ka \\ kb \end{pmatrix}$$

That is, each component of the vector is multiplied by the number.

For example,

$$2\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2(-3) \\ 2(4) \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$$

5.14.2 The length of a vector

Given a vector $\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$, the length of $\mathbf{v}$, denoted $|\mathbf{v}|$, is given by the formula

$$|\mathbf{v}| = \sqrt{x^2 + y^2}$$

For example, the length of the vector $\mathbf{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ is

$$|\mathbf{a}| = \left|\begin{pmatrix} -2 \\ 3 \end{pmatrix}\right| = \sqrt{(-2)^2 + 3^2} = \sqrt{4+9} = \sqrt{13} \text{ units}$$

5.15 Trigonometry

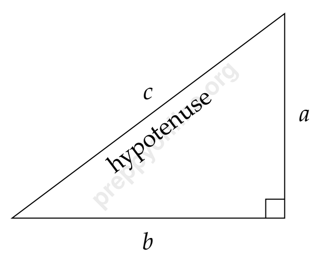
Trigonometry is about the measurement of the sides and angles of triangles.

Right-angled triangles are of special interest here because, the the basic trigonometric ratios (sin, cos, tan) are easy to define in terms of the sides of such a triangle.

5.15.1 Pythagoras' theorem

Given a right angled triangle like the one below, the longest side is called they hypotenuse.

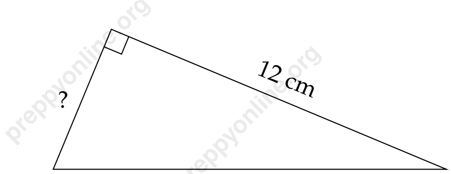
The hypotenuse is always opposite the 90° angle in the triangle.



Pythagoras' theorem states that, if the length of the hypotenuse is c and the lengths of the other sides are a and b, then the relationship between the lengths is

$$c^2 = a^2 + b^2$$

For example, since the triangle below is a right-angled triangle with hypotenuse of length 13 cm, we can use Pythagoras' theorem to find the third side as follows.



Let the length of the third side be x. Then, by Pythagoras' theorem,

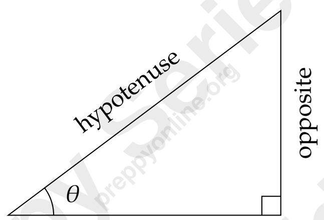
$$\begin{aligned} x^2 + 5^2 &= 13^2 \\ x^2 + 25 &= 169 \\ x^2 &= 169 - 25 \\ x^2 &= 144 \\ x &= 12 \end{aligned}$$

Thus, the length of the third side is 12 cm.

5.15.2 SOH-CAH-TOA

When one of the angles in a right-angled triangle is of interest, we use that as a reference angle to name the sides of the triangle.

The side opposite the reference angle is called the opposite side, while the side next to the reference angle (that is not the hypotenuse) is called the adjacent side. The longest side, the hypotenuse, remains the hypotenuse.



With these labels, the basic trigonometric ratios as follows.

The sine of θ is given by

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

The cosine of θ is given by

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

The tangent of θ is given by

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

These are shortened by the mnemonic SOH-CAH-TOA.

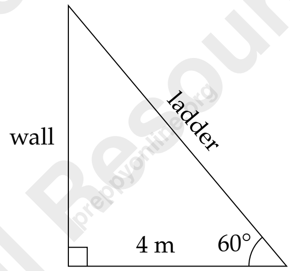
SOH means $\text{S} = \frac{\text{O}}{\text{H}}$, where S stands for sin, O stands for "opposite," and H stands for "hypotenuse."

CAH means $\text{C} = \frac{\text{A}}{\text{H}}$, where C stands for cos, A stands for "adjacent," and H stands for "hypotenuse."

TOA means $\text{T} = \frac{\text{O}}{\text{A}}$, where T stands for tan, O stands for "opposite," and A stands for "adjacent."

A ladder leans against a vertical wall so that it makes an angle of 60° with the ground. If the foot of the ladder is 4 metres from the wall, find the length of the ladder.

The sketch below shows the relationship between the ladder, the wall, and the floor.



Let the length of the ladder be x. Then, from the sketch,

$$\cos 60^\circ = \frac{4\text{ m}}{x}$$

Thus,

$$x = \frac{4\text{ m}}{\cos 60^\circ}$$

If we know the value of $\cos 60^\circ$, we can use it to calculate the length of the ladder.

Using a calculator, we find $\cos 60^\circ$ to be $\frac{1}{2}$. Hence, the length of the ladder is

$$\frac{4\text{ m}}{\frac{1}{2}} = 2(4\text{ m}) = 8\text{ m}$$

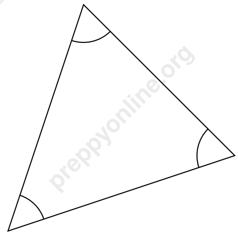
5.16 Plane geometry

5.16.1 Types of angles

Acute angles

An acute angle is an angle that measures greater than 0° and less than 90°.

Acute angles are known for their sharpness or pointedness.

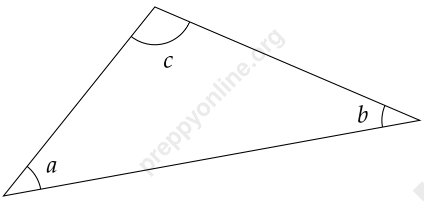


All the angles in the triangle above are examples of acute angles.

Obtuse angles

An obtuse angle is an angle that measures greater than 90° and less than 180°.

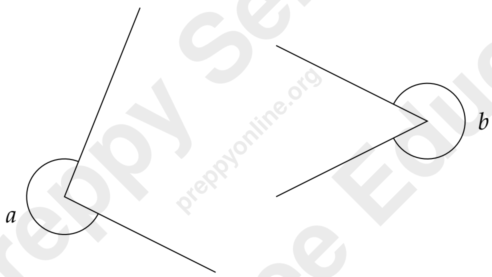
Angle c below is an obtuse angle, while angles a and b are acute angles.



Reflex angles

A reflex angle is an angle that measures greater than 180° and less than 360°.

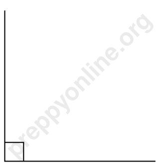
Examples of reflex angles are angles a and b below.



Right angles

A right angle is an angle that measures exactly 90°.

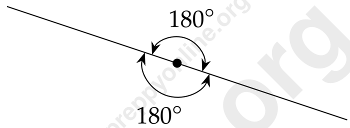
It is the angle at the corner of a square or a perfect "L" shape.



Straight angles

A straight angle is an angle that measures 180°.

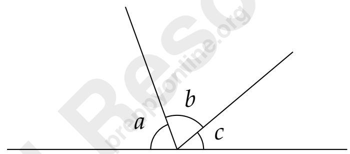
They are angles about a point on either side of a straight line.



Angles on a straight line

Since a straight angle measures exactly 180°, when it is divided up, the measures of the parts sum up to 180°.

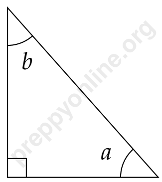
For example, angles a, b, and c add up to 180° as they are angles on a straight line.



Complementary angles

Two angles are called complementary angles if they add up to 90°.

An example of complementary angles is the two acute angles in a right-angled triangle.



Since the angles in a triangle add up to 180°,

$$a + b + 90° = 180°$$

Hence,

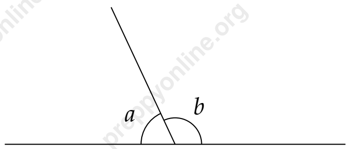
$$a + b = 180° - 90° = 90°$$

Therefore, a and b are complementary angles.

Supplementary angles

Supplementary angles are a pair of angles whose measures add up to 180°.

An example of supplementary angles are two angles on a straight line like angles a and b below.

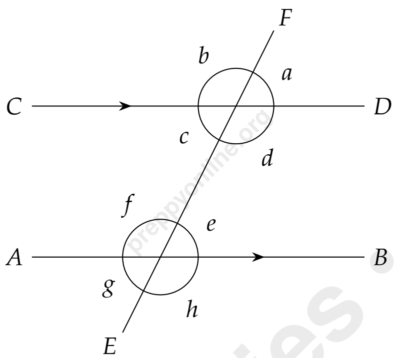


5.16.2 Angles in parallel lines

In the diagram below, lines AB and CD are parallel, as indicated by the arrows on them.

Line EF cuts through lines AB and CD. It is called a transversal.

A transversal is a line that crosses two or more lines in a plane.



Whenever a transversal crosses a pair of parallel lines, it creates 8 angles with special relationships between them.

We shall discuss the relationships between the 8 angles a, b, c, d, e, f, g, h below.

Alternate angles

Angles on opposite sides of the transversal and between the parallel lines are equal.

These are called alternate angles.

In the diagram above, c and e are alternate angles. So are d and f.

Corresponding angles

Angles in the same relative position at each intersection are equal.

For example, the angle at the top-left of the intersection at the top, b, is equal to the angle at the top-left of the intersection at the bottom, f.

Co-interior angles

Angles on the same side of the transversal and between the parallel lines add up to 180°.

These are called co-interior or allied angles. They are also called consecutive interior angles.

Examples are angles d and e in the diagram.

c and f are also examples of co-interior angles.

Vertically opposite angles

Angles opposite each other at the intersections are equal.

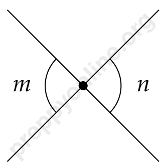
These are called vertically opposite angles.

Examples include a and c in the diagram above.

Angles b and d are also vertically opposite and therefore equal.

Vertically opposite angles are called "vertically" opposite because they are opposite each other across their common vertex—the point where two lines meet or cross—and not because they are oriented up and down. Thus, here, "vertical" is the adjectival form of "vertex."

In the diagram below, angles m and n are vertically opposite angles even though they are not oriented up and down.



5.17 Statistics

5.17.1 Types of data

Quantitative data

Quantitative data are values that can be measured numerically. Examples include

- length (as it can be measured in numbers like metres, centimetres, inches, etc.)
- weight (as it can be measured in numbers like kilograms, pounds, etc.)
- time (as it can be measured in numbers like hours, minutes, seconds, etc.)
- speed (as it can be measured in numbers like metres per second, kilometres per hour, etc.)
- temperature (as it can be measured in numbers like degrees Celsius, degrees Fahrenheit, etc.)

Qualitative data

Qualitative data are values that describe qualities or attributes rather than numerical measurements. Such data can typically be arranged into categories or groups. Examples include

- colour (red, blue, green, etc.)
- marital status (single or married)
- nationality (Ghanaian, Nigerian, Japanese, etc.)

5.17.2 Measures of central tendency

These give a single number by which the data can be represented. Examples include the mean, the median, and the mode.

The mean

The mean of a list of numbers is given by the sum of the numbers divided by how many numbers there are.

For example, the mean of the numbers $1, 1, 2, 2, 2, 2, 3, 3, 4$ is

$$\frac{1+1+2+2+2+2+3+3+4}{9} = \frac{20}{9}$$

The median

The median is the middle number when data is arranged in ascending or descending order.

For example, in the data $1, 1, 2, 2, 2, 2, 3, 3, 4$, the median is 2 because 2 is the middle number when the numbers are arranged in ascending or descending order.

The mode

Given some data, such as a list of numbers, the mode is the data that occurs most frequently.

For example, in the data $1, 1, 2, 2, 2, 2, 3, 3, 4$,

1 appears 2 times.

2 appears 4 times.

3 appears 2 times.

4 appears once.

Hence, the mode is 2 as 2 is the data that appears most frequently.

5.17.3 Measures of dispersion

These give an idea of spread out the data is.

Examples include the range and the standard deviation.

Range

Given a list of numbers, the range is the difference between the largest and the smallest.

For example, in the list $1, 1, 2, 2, 2, 2, 3, 3, 4$, the range is

$$4 - 1 = 3$$

since the largest number in the list is 4 and the smallest number is 1.

5.18 Loci

A locus is a path traced by a moving point.

There are many rules by which a point could move, and each of them gives a locus.

We shall mention some important examples below.

5.18.1 Examples of loci

The circle

If a point moves so that it is always the same distance from a given point, the moving point will trace a circle with the given point as its centre.

The perpendicular bisector

The set of points that are equidistant from two given points will trace the perpendicular bisector of the line joining the two given points.

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