Preppy Series

ELECTIVE MATHEMATICS

for Senior High Schools

WASSCE Past Questions & Solutions

Arranged by Year

preppyonline.org

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Sample Edition

Preface

This book is designed to be a helpful resource for senior high school students, especially those preparing for the West African Senior School Certificate Examination (WASSCE) of the West African Examinations Council (WAEC).

It features

- a collection of complete WASSCE Elective/Further Mathematics past papers—both objective and essay-type questions—arranged by year
- detailed and thoughtfully explained solutions
- short notes on important facts and methods frequently used on the exams

Our philosophy is simple: learning Mathematics should go beyond memorizing procedures or blindly applying algorithms and tricks. True understanding comes from engaging with problems, exploring different approaches, and appreciating how and why solutions work. For this reason, each solution is presented

- in a step by step fashion
- with clear reasoning
- with multiple perspectives where appropriate
- and with an emphasis on building strong conceptual foundations

Special attention has been given to clarity, with well-drawn diagrams and structured workings that carry the student along through the solution process.

This free sample is available at preppyonline.org. The full version, including complete detailed solutions for both objective and essay-type questions, is available for purchase from the same website.

It is our hope that students, teachers, and independent learners alike will find it useful not just for exam preparation, but for developing confidence and deeper insight into Mathematics.

Comments, suggestions, and other enquiries may be sent to preppyonline@gmail.com

Contents

Chapter 1

2026 Paper 1

1. Given that $x + \sqrt{3x+4} = 8$, find the value of x.

A. 4
B. 3
C. 2
D. 1

2. Given that $\frac{{}^{n}P_6}{{}^{n}C_6} = 12x$, find the value of x.

A. 40
B. 60
C. 480
D. 720

3. Find the remainder when $4x^3 - 5x + 3$ is divided by $2x - 1$.

A. $\frac{1}{2}$
B. 1
C. 2
D. 3

4. Solve: $\sin x - \cos x = 0$, where $0° \leq x \leq 180°$.

A. 180°
B. 135°
C. 90°
D. 45°

5. If the midpoint of $X(4, -1)$ and $Z(m, n)$ is $P(3, -4)$, find the coordinates of Z.

A. $(-10, -7)$
B. $(2, -9)$
C. $(2, -7)$
D. $(-10, -9)$

6. If $\begin{vmatrix} 4 & 4 \\ 4 & 6 \end{vmatrix} = b$, evaluate $b^{\frac{1}{3}}$.

A. 1
B. 2
C. 4
D. 6

7. Solve: $3^{5x-1} - 2 = 0$.

A. 0.23
B. 0.33
C. 1.13
D. 1.32

8. The first and the last terms of an **Arithmetic Progression (A.P.)** are 7 and 70 respectively. If the sum is 385, find the number of terms in the series.

A. 12
B. 10
C. 8
D. 5

9. If $f'(x) = 4x^3 - 6x + 1$ and $f(-1) = 0$, find $f(x)$.

A. $f(x) = x^4 - 3x^2 + x - 5$
B. $f(x) = x^4 - 3x^2 + x + 5$
C. $f(x) = x^4 - 3x^2 + x - 3$
D. $f(x) = x^4 - 3x^2 + x + 3$

10. The deviations of a set of data from the mean are $-7, -5, -k, 2k$ and $3k$. Find the value of k.

A. -4
B. -3
C. 3
D. 4

11. Given that $\frac{18}{y^2} + \frac{1}{y} = 4$, find the positive value of y.

A. $\frac{4}{9}$
B. $\frac{2}{3}$
C. $\frac{3}{2}$
D. $\frac{9}{4}$

12. There are 6 passengers in a bus. In how many ways will 4 passengers alight one after the other at 4 different bus stops?

A. 720
B. 516
C. 480
D. 360

13. Find, in terms of π, the area of the circle $x^2 + y^2 + 4x - 6y + 3 = 0$.

A. 13π square units
B. 10π square units
C. 9π square units
D. 3π square units

14. Given that $\frac{R - 29x}{2x^2 + 3x - 35} \equiv \frac{5}{(2x-7)} - \frac{17}{(x+5)}$, find the value of R.

A. 50
B. 72
C. 100
D. 144

15. Consider the propositions:

x: Stephen is good at Biology.
w: Stephen is good at Health Education.

Write in symbolic form, the statement "Stephen is not good at both Health Education and Biology".

A. $\sim (x \wedge w)$
B. $\sim x \wedge w$
C. $\sim x \vee w$
D. $\sim (x \vee w)$

16. If $\log_2 x = \frac{2}{3}$ and $\log_2 y = \frac{3}{2}$, find the value of $\log_2 \sqrt{\frac{y^4}{x^3}}$.

A. 1
B. 2
C. 3
D. 4

17. If the domain of $f : x \to \frac{1}{2+x}$ is $\{1, 2, 3, 4, 5\}$, find the range.

A. $\frac{1}{3} \leq f(x) \leq \frac{1}{7}$
B. $\frac{1}{7} \leq f(x) \leq \frac{1}{3}$
C. $-\frac{1}{7} \leq f(x) \leq -\frac{1}{3}$
D. $-\frac{1}{7} \leq f(x) \leq \frac{1}{3}$

18. Given that $\mathbf{p} = 3\mathbf{i} - 7\mathbf{j}$, $\mathbf{q} = 4\mathbf{i} - 9\mathbf{j}$ and $\mathbf{r} = -2\mathbf{i} + 5\mathbf{j}$. Find $|3\mathbf{p} - 2\mathbf{q} + \mathbf{r}|$.

A. 8.26
B. 7.00
C. 6.26
D. 2.24

19. Which of the following matrices does **not** have an inverse?

A. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
B. $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$
C. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
D. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

20. Factorize: $8p^3 - 27$.

A. $(2p-3)(4p^2+6p+9)$
B. $(2p-3)(4p^2-6p+9)$
C. $(2p+3)(4p^2+6p-9)$
D. $(2p+3)(4p^2-6p-9)$

21. If $(1+2x)^n \equiv \sqrt{0.98}$, what are the values of x and n?

A. $x = -0.01, n = \frac{1}{2}$
B. $x = -0.01, n = -\frac{1}{2}$
C. $x = 0.01, n = -\frac{1}{2}$
D. $x = 0.01, n = \frac{1}{2}$

22. If $\mathbf{p} = 2\mathbf{i} - 7\mathbf{j}$, $\mathbf{q} = x\mathbf{i} + y\mathbf{j}$ and $\mathbf{p} + 3\mathbf{q} = \mathbf{i} - 5\mathbf{j}$, find the values of x and y.

A. $x = -\frac{1}{3}$ and $y = -\frac{2}{3}$
B. $x = \frac{1}{3}$ and $y = -\frac{2}{3}$
C. $x = -\frac{1}{3}$ and $y = \frac{2}{3}$
D. $x = \frac{1}{3}$ and $y = \frac{2}{3}$

23. If $f(x+1) = 2x^2 - 4x + 2$, find $f(2)$.

A. 0
B. 6
C. 8
D. 10

24. Find the minimum value of $y = x^2 + 9x - 17$.

A. $\frac{149}{4}$
B. $\frac{49}{4}$

C. $-\frac{49}{4}$

D. $-\frac{149}{4}$

25. Given that $(1+\sqrt{3})y - x\sqrt{3} = 5$, find the slope of the line.

A. $-\frac{3}{2}+\frac{1}{2}\sqrt{3}$

B. $-\frac{3}{2}-\frac{1}{2}\sqrt{3}$

C. $\frac{3}{2}+\frac{1}{2}\sqrt{3}$

D. $\frac{3}{2}-\frac{1}{2}\sqrt{3}$

26. Given that P is the complement of Q, where P and Q are subsets of the universal set μ, which of the following is **not** true?

A. $(P' \cup Q') = P \cap Q$

B. $(P \cap Q)' = P \cup Q$

C. $P \cap Q = \phi$

D. $P \cup Q = \mu$

27. A particle is acted upon by the forces $\mathbf{F}_1 = \begin{pmatrix}7\\3\end{pmatrix}$ N, $\mathbf{F}_2 = \begin{pmatrix}3\\-1\end{pmatrix}$ N and $\mathbf{F}_3 = \begin{pmatrix}-2\\5\end{pmatrix}$ N. Find the force required to keep the particle in equilibrium.

A. $\begin{pmatrix}-12\\-7\end{pmatrix}$

B. $\begin{pmatrix}-8\\-7\end{pmatrix}$

C. $\begin{pmatrix}8\\7\end{pmatrix}$

D. $\begin{pmatrix}12\\7\end{pmatrix}$

28. Calculate the coefficient of x^4y^3 in the binomial expansion of $\left(2x - \frac{y}{2}\right)^7$.

A. -70

B. -35

C. 35

D. 70

29. The operation Δ is defined on the set $T = \{p, q, r\}$ by the table:

Δ	p	q	r
p	r	p	q
q	p	q	r
r	q	r	p

Find the inverse of the element q under Δ.

A. $\frac{1}{r}$

B. r

C. q

D. p

30. A force of 40 N acts on a particle of mass 8 kg which is at rest. How far will the particle move in 4 s?

A. 10 m

B. 32 m

C. 40 m

D. 60 m

31. Simplify: $[(1+\sin x)(1-\sin x)]^{\frac{1}{2}}$.

A. $\cos^2 x$

B. $\sin^2 x$

C. $\sin x$

D. $\cos x$

32. Kofi attended a selection interview for three jobs. There were 3, 4 and 2 candidates for the first, second and third jobs respectively. Find the probability that he will be selected for **only one** of the jobs.

A. $\frac{5}{8}$

B. $\frac{7}{12}$

C. $\frac{11}{24}$

D. $\frac{1}{8}$

33. If $\frac{y}{\sqrt{3}-1} = 3+3\sqrt{3}$, find the value of y.

A. 9

B. 6

C. 3

D. 2

34. Evaluate: $\lim_{x\to 5}\left(\frac{x^2-25}{x-5}\right)$.

A. 25

B. 10

C. 5

D. 0

35. Calculate the mean deviation of 3, 5, 8, 11, 12 and 21.

A. 4.67

B. 3.80

C. 2.40

D. 1.40

36. Find the magnitude of the resultant of the forces $P(12N, 35°)$ and $R(8N, 210°)$.

A. 4.06 N

B. 4.07 N

C. 4.08 N

D. 4.09 N

37. The probabilities that Kofi and Adams hit a target in a shooting competition are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. What is the probability that **only one** of them will hit the target?

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{9}$

38. If $h = \int (3t^2 + 2t)\,dt$ and $h = 4$ when $t = 1$, find h.

A. $h = t^3 + t^2 - 6$

B. $h = t^3 + t^2 + 6$

C. $h = t^3 + t^2 - 2$

D. $h = t^3 + t^2 + 2$

39. Express the recurring decimal 0.636363... as a single fraction.

A. $\frac{7}{11}$

B. $\frac{7}{10}$

C. $\frac{7}{9}$

D. $\frac{7}{8}$

40. A particle of mass 4 kg at rest is acted upon by forces $(2\mathbf{i} - 3\mathbf{j})$ and $(-6\mathbf{i} + 11\mathbf{j})$. Find its velocity in 5 seconds.

A. $-5\mathbf{i} - 10\mathbf{j}$

B. $-5\mathbf{i} + 10\mathbf{j}$

C. $5\mathbf{i} - 10\mathbf{j}$

D. $5\mathbf{i} + 10\mathbf{j}$

Chapter 2

Solutions to 2026 Paper 1

Answer key

1. A	9. D	17. X	25. D	33. B
2. B	10. C	18. D	26. A	34. B
3. B	11. D	19. B	27. B	35. A
4. D	12. D	20. A	28. A	36. D
5. C	13. B	21. A	29. C	37. A
6. B	14. D	22. C	30. C	38. D
7. B	15. A	23. A	31. D	39. A
8. B	16. B	24. D	32. C	40. B

Solutions

1. Answer: A

Because this is a question in an objective test, we can proceed by trial and error, substituting each of the given values into the equation and checking if it holds true. Alternatively, we can solve the equation algebraically.

We shall use both methods below.

Method 1: Trial and error

Substituting $x = 4$ into the equation gives

$$4 + \sqrt{3(4)+4} = 4 + \sqrt{12+4} = 4 + \sqrt{16} = 4 + 4 = 8$$

Thus, $x = 4$ satisfies the equation.

Luckily for us, the first option we tried ($x = 4$) satisfies the equation so, in an exam, we could move on to the next question. However, let's check the other options for completeness.

Substituting $x = 3$ into the equation gives

$$3 + \sqrt{3(3)+4} = 3 + \sqrt{9+4} = 3 + \sqrt{13} \neq 8$$

Substituting $x = 2$ into the equation gives

$$2 + \sqrt{3(2)+4} = 2 + \sqrt{6+4} = 2 + \sqrt{10} \neq 8$$

Substituting $x = 1$ into the equation gives

$$1 + \sqrt{3(1)+4} = 1 + \sqrt{3+4} = 1 + \sqrt{7} \neq 8$$

From our trial and error, the value of x that satisfies the equation is 4.

Method 2: Algebraic solution

First, we isolate the square root term on one side of the equation, and then square both sides to eliminate the square root.

$$\begin{aligned}
x + \sqrt{3x+4} &= 8 \\
\sqrt{3x+4} &= 8 - x \\
3x + 4 &= (8-x)^2 \\
3x + 4 &= 64 - 16x + x^2 \\
x^2 - 19x + 60 &= 0 \\
(x-4)(x-15) &= 0 \\
x &= 4 \text{ or } 15
\end{aligned}$$

We must check both solutions in the original equation because squaring both sides of an equation can introduce extraneous solutions.

When $x = 4$, we get

$$\begin{aligned}
4 + \sqrt{3(4)+4} &= 4 + \sqrt{12+4} \\
&= 4 + \sqrt{16} \\
&= 4 + 4 \\
&= 8
\end{aligned}$$

Thus, $x = 4$ satisfies the equation.

When $x = 15$, we get

$$\begin{aligned}
15 + \sqrt{3(15)+4} &= 15 + \sqrt{45+4} \\
&= 15 + \sqrt{49} \\
&= 15 + 7 \\
&= 22
\end{aligned}$$

Thus, $x = 15$ does not satisfy the equation.

Therefore, the only solution is $x = 4$.

2. Answer: B

We shall look at two ways of solving this problem. The first one is algebraic while the second is combinatorial.

In the algebraic solution, we write out the algebraic expressions representing ${}^nP_6$ and ${}^nC_6$ and then simplify. But in the combinatorial approach, we use the relationship between ${}^nP_r$ and ${}^nC_r$.

Method 1: Algebraic solution

Recall that ${}^nP_r = \dfrac{n!}{(n-r)!}$ and ${}^nC_r = \dfrac{n!}{r!(n-r)!}$.

Hence,

$$\begin{aligned}\frac{{}^nP_6}{{}^nC_6} &= \frac{\frac{n!}{(n-6)!}}{\frac{n!}{6!(n-6)!}}\\ &= \frac{n!}{(n-6)!}\cdot\frac{6!(n-6)!}{n!}\\ &= 6!\end{aligned}$$

Setting this equal to $12x$ gives

$$\begin{aligned}12x &= 6!\\ 12x &= 720\\ x &= \frac{720}{12}\\ x &= 60\end{aligned}$$

Method 2: Combinatorial solution

The relationship between ${}^nP_r$ and ${}^nC_r$ is

$${}^nP_r = r! \times {}^n C_r$$

This is because ${}^nP_r$ represents the number of ordered arrangements (permutations) of r objects from a set of n objects, while ${}^nC_r$ represents the number of unordered arrangements (combinations) of r objects from a set of n objects, and every ordered arrangement of r objects can be obtained by first choosing r objects from a set of n objects in ${}^nC_r$ ways, and then arranging the r objects in $r!$ ways.

Thus,

$$\frac{{}^nP_6}{{}^nC_6} = 6!$$

Setting this equal to $12x$ gives

$$\begin{aligned}12x &= 6!\\ 12x &= 720\\ x &= \frac{720}{12}\\ x &= 60\end{aligned}$$

3. Answer: B

We will use three methods to solve this: the remainder theorem, polynomial long division, and synthetic division.

Method 1: Remainder theorem

According to the remainder theorem, if a polynomial $f(x)$ is divided by $ax - b$, the remainder is $f\left(\frac{b}{a}\right)$. In this case, $f(x) = 4x^3 - 5x + 3$ and the divisor is $2x - 1$. So, the remainder is $f\left(\frac{1}{2}\right)$.

$$\begin{aligned}f\left(\frac{1}{2}\right) &= 4\left(\frac{1}{2}\right)^3 - 5\left(\frac{1}{2}\right) + 3\\ &= 4\left(\frac{1}{8}\right) - \frac{5}{2} + 3\\ &= \frac{4}{8} - \frac{5}{2} + 3\\ &= \frac{1}{2} - \frac{5}{2} + 3\\ &= \frac{1-5}{2} + 3\\ &= \frac{-4}{2} + 3\\ &= -2 + 3\\ &= 1\end{aligned}$$

Method 2: Polynomial long division

We perform polynomial long division to find the remainder when $4x^3 - 5x + 3$ is divided by $2x - 1$.

$$\begin{array}{r l}
 & 2x^2 + x - 2\\ \cline{2-2}
2x-1\,) & 4x^3 \qquad\quad - 5x + 3\\
 & -\,4x^3 + 2x^2\\ \cline{2-2}
 & \qquad 2x^2 - 5x\\
 & \quad -\,2x^2 + x\\ \cline{2-2}
 & \qquad\qquad -4x + 3\\
 & \qquad\qquad\ \ 4x - 2\\ \cline{2-2}
 & \qquad\qquad\qquad 1
\end{array}$$

The remainder is 1.

Chapter 3

2026 Paper 2

Section A

48 marks

Answer all the questions in this section.

All questions carry equal marks.

1. Given that $P = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$, $Q = \{x : -7 < x < 7\}$ and $R = \{x : x > 0\}$ are subsets of the universal set $\mu = \{x : x \in Z\}$, find:

(a) $P \cap R$;

(b) $P' \cap Q$;

(c) $n(Q \cap R)$.

2. A binary operation $*$ is defined on the set of real numbers, R, by $p * q = \dfrac{3p + 2q}{2p + 3q}$, where $p, q \in R$.

(a) Evaluate $\left(\sqrt{3}\right) * \left(\sqrt{2}\right)$.

(b) Find the values of x for which $x * 3 = \dfrac{1}{2}$.

3. Solve: $2\sqrt{(2x-5)} - \sqrt{(4x-3)} = 1$.

4. The function g is defined on the set, R, of real numbers by $g : x \rightarrow \dfrac{5x+7}{3x+2}$, $x \neq -\dfrac{2}{3}$. Find the value of x if $g^{-1}(x) = g(-2)$.

5. The income (x) and food expenditure (y) of ten families were randomly selected from a particular village in Ghana. The data yielded the following results: $\sum x^2 = 9{,}577$, $\sum x = 293$, $\sum y^2 = 701$, $\sum y = 81$, $\sum xy = 2{,}574$, $\bar{x} = 29.3$ and $\bar{y} = 8.1$. Estimate the equation of the line of best-fit using *least squares method* of food expenditure (y) on income (x).

6. Two events X and Y are such that $P(X) = \dfrac{1}{3}$, $P(Y) = \dfrac{11}{20}$ and $P(X \cap Y) = \dfrac{9}{40}$. Find:

(a) $P(X \cap Y')$;

(b) $P(X \cup Y)$.

7. The position vectors of M, N and R from the origin, O, are m, n and r respectively. If OMN is a triangle and R is a point on $\overrightarrow{MN}$ such that $\overrightarrow{MR} = \dfrac{1}{4}\overrightarrow{RN}$, find an expression for r in terms of m and n.

8. A see-saw pivoted at the middle is kept in balance by the masses of Kofi, Mensah and Dela such that only Kofi, whose mass is 60 kg, sits on one side. If they sit at distances 2 m, 3 m, and 4 m respectively from the pivot and Dela is 15 kg, find the mass of Mensah.

Section B

52 marks

Answer four questions only from this section with at least one question from each part.

All questions carry equal marks.

Part I: Pure Mathematics

9. If α and β are the non-zero roots of $3x^2 - 11x + 4 = 0$, find the equation whose roots are $\left(\alpha^2 + \dfrac{1}{\beta}\right)$ and $\left(\dfrac{1}{\alpha} + \beta^2\right)$.

10. The gradient of a curve which passes through $P(2, 7)$ is given by $\dfrac{dy}{dx} = \dfrac{15}{\sqrt{(7x+2)}}$. Find the equation of the:

(a) curve;

(b) normal to the curve at P.

11. The points $(-3, 5)$ and $(7, 9)$ are the ends of the diameter of a circle. Find the equation of the:

(a) circle;

(b) normal to the circle at $(3, 4)$.

Part II: Statistics and Probability

12. The table shows the distribution of marks obtained by some candidates in an examination.

Marks	0 - 9	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 - 89	90 - 99
Freq.	2	6	8	15	28	21	9	7	3	1

(a) (i) construct a cumulative frequency table for the distribution;

(ii) use your table in **12(a)(i)** to draw the cumulative frequency curve (ogive).

(b) Use your graph (ogive) in **12(a)(ii)** to estimate, correct to **one** decimal place, the:

(i) upper quartile of the distribution;

(ii) lowest mark for distinction, if 18% of the candidates pass with distinction.

13. A factory produces matchsticks of which 20% are defective. If 12 matchsticks are randomly selected with replacement, from a box, find the probability that:

(a) 75% are **not** defective;

(b) $33\frac{1}{3}$% are defective;

(c) number of defective and non-defective matchsticks **are equal**.

Part III: Vectors and Mechanics

14. Three forces $9\sqrt{3}$ N, $12\sqrt{5}$ N and 18 N act along the vectors $(i + 3j)$, $(2i - \sqrt{3}j)$ and $(i + \sqrt{5}j)$ respectively. Calculate, correct to the **nearest** whole number, the magnitude and direction of their resultant force.

15. A 20 kg box is placed on a smooth plane inclined at 30° to the horizontal. If the box starts moving from rest, find, correct to **one** decimal place, the:

(a) velocity of the box after sliding 6 m from the starting point;

(b) momentum of the box 6 m from the starting point.

[Take $g = 10\ \text{ms}^{-2}$]

Chapter 4

Solutions to 2026 Paper 2

Question 1

We are given the following:

$P = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$

$Q = \{x : -7 < x < 7\}$

$R = \{x : x > 0\}$

P, Q, and R are subsets of the universal set $\mu = \{x : x \in Z\}$.

(a) $P \cap R$ is the set of elements that are in both set P and set R.

Since R is the set of positive integers, and P is the set of even integers, $P \cap R$ is the set of positive even integers.

So, $P \cap R = \{2, 4, 6, \ldots\}$.

(b) $P' \cap Q$ is the set of elements that are in the complement of set P and set Q.

Since P is the set of even integers, P' is the set of odd integers.

So, since Q is the set of integers between -7 and 7, $P' \cap Q$ is the set of odd integers between -7 and 7.

Thus, $P' \cap Q = \{-5, -3, -1, 1, 3, 5\}$.

(c) $n(Q \cap R)$ is the number of elements in the intersection of set Q and set R.

Since R is the set of positive integers, and Q is the set of integers between -7 and 7, $Q \cap R$ is the set of positive integers between -7 and 7.

So, $Q \cap R = \{1, 2, 3, 4, 5, 6\}$, and $n(Q \cap R) = 6$.

Question 2

(a) To evaluate $\left(\sqrt{3}\right) * \left(\sqrt{2}\right)$, we substitute $p = \sqrt{3}$ and $q = \sqrt{2}$ into the given formula:

$$\left(\sqrt{3}\right) * \left(\sqrt{2}\right) = \frac{3\sqrt{3} + 2\sqrt{2}}{2\sqrt{3} + 3\sqrt{2}}$$

To simplify this expression, we rationalize the denominator by multiplying the numerator and the denominator by the conjugate of the denominator, which is $2\sqrt{3} - 3\sqrt{2}$:

$$\begin{aligned}
\left(\sqrt{3}\right) * \left(\sqrt{2}\right) &= \frac{3\sqrt{3} + 2\sqrt{2}}{2\sqrt{3} + 3\sqrt{2}} \\
&= \frac{3\sqrt{3} + 2\sqrt{2}}{2\sqrt{3} + 3\sqrt{2}} \times \frac{2\sqrt{3} - 3\sqrt{2}}{2\sqrt{3} - 3\sqrt{2}} \\
&= \frac{\left(3\sqrt{3} + 2\sqrt{2}\right)\left(2\sqrt{3} - 3\sqrt{2}\right)}{\left(2\sqrt{3}\right)^2 - \left(3\sqrt{2}\right)^2} \\
&= \frac{3\sqrt{3}(2\sqrt{3}) - 3\sqrt{3}(3\sqrt{2}) + 2\sqrt{2}(2\sqrt{3}) - 2\sqrt{2}(3\sqrt{2})}{4(3) - 9(2)} \\
&= \frac{18 - 9\sqrt{6} + 4\sqrt{6} - 12}{12 - 18} \\
&= \frac{6 - 5\sqrt{6}}{-6} \\
&= \frac{5\sqrt{6} - 6}{6}
\end{aligned}$$

(b) To find the values of x for which $x * 3 = \frac{1}{2}$, we substitute $p = x$ and $q = 3$ into the given formula, set it equal to $\frac{1}{2}$ and solve the resulting equation.

$$x * 3 = \frac{3x + 2(3)}{2x + 3(3)} = \frac{3x + 6}{2x + 9}$$

Setting this expression equal to $\frac{1}{2}$ and solving, we get

$$\begin{aligned}\frac{3x+6}{2x+9} &= \frac{1}{2}\\ 2(3x+6) &= 1(2x+9)\\ 6x+12 &= 2x+9\\ 6x-2x &= 9-12\\ 4x &= -3\\ x &= -\frac{3}{4}\end{aligned}$$

Thus, the only value of x for which $x * 3 = \frac{1}{2}$ is $-\frac{3}{4}$.

Question 3

To solve $2\sqrt{(2x-5)} - \sqrt{(4x-3)} = 1$, we rearrange the equation to get only one square root term on each side of the equation. We do this to reduce the amount of work we have to do when we square both sides of the equation.

$$\begin{aligned}2\sqrt{2x-5} - \sqrt{4x-3} &= 1\\ 2\sqrt{2x-5} &= 1 + \sqrt{4x-3}\end{aligned}$$

Squaring both sides of the equation, we get

$$\begin{aligned}\left(2\sqrt{2x-5}\right)^2 &= \left(1+\sqrt{4x-3}\right)^2\\ 4(2x-5) &= 1^2 + 2(1)\sqrt{4x-3} + \left(\sqrt{4x-3}\right)^2\\ 8x-20 &= 1 + 2\sqrt{4x-3} + 4x - 3\\ 8x-20 &= 4x-2+2\sqrt{4x-3}\\ 4x-18 &= 2\sqrt{4x-3}\\ 2x-9 &= \sqrt{4x-3}\end{aligned}$$

We square both sides of the equation again to get

$$\begin{aligned}(2x-9)^2 &= \left(\sqrt{4x-3}\right)^2\\ 4x^2-36x+81 &= 4x-3\\ 4x^2-40x+84 &= 0\\ x^2-10x+21 &= 0\\ (x-3)(x-7) &= 0\\ x = 3 \quad &\text{or} \quad x = 7\end{aligned}$$

Squaring both sides of an equation can introduce extraneous solutions, so we must check our solutions against the original equation.

For $x = 3$, we have

$$\begin{aligned}2\sqrt{(2(3)-5)} - \sqrt{(4(3)-3)} &= 2\sqrt{(6-5)} - \sqrt{(12-3)}\\ &= 2\sqrt{1} - \sqrt{9}\\ &= 2(1) - 3\\ &= -1\end{aligned}$$

Since $-1 \neq 1$, $x = 3$ is not a valid solution.

For $x = 7$, we have

$$\begin{aligned}2\sqrt{(2(7)-5)} - \sqrt{(4(7)-3)} &= 2\sqrt{(14-5)} - \sqrt{(28-3)}\\ &= 2\sqrt{9} - \sqrt{25}\\ &= 2(3) - 5\\ &= 6-5\\ &= 1\end{aligned}$$

Since $1 = 1$, $x = 7$ is a valid solution.

Thus, the solution to the equation is $x = 7$.